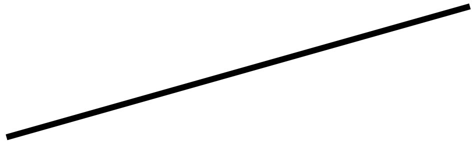
Computergrafik SS 2010
Oliver Vornberger

noch Kapitel 3:
2D-Grundlagen

Vorlesung vom 13.04.10

Algorithmen zum Zeichnen



Parametrisiert:

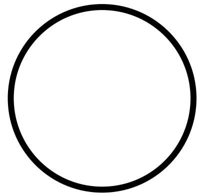
$x := f_1(t); \; y := f_2(t)$

Gradengleichung:

$y := f(x)$

Bresenham:

x++; if (…) {y++; … }



Parametrisiert:

$x = f_1(t); \; y := f_2(t)$

Kreisgleichung:

$y := f(x)$

Bresenham:

x++; if (…) {y--; … }

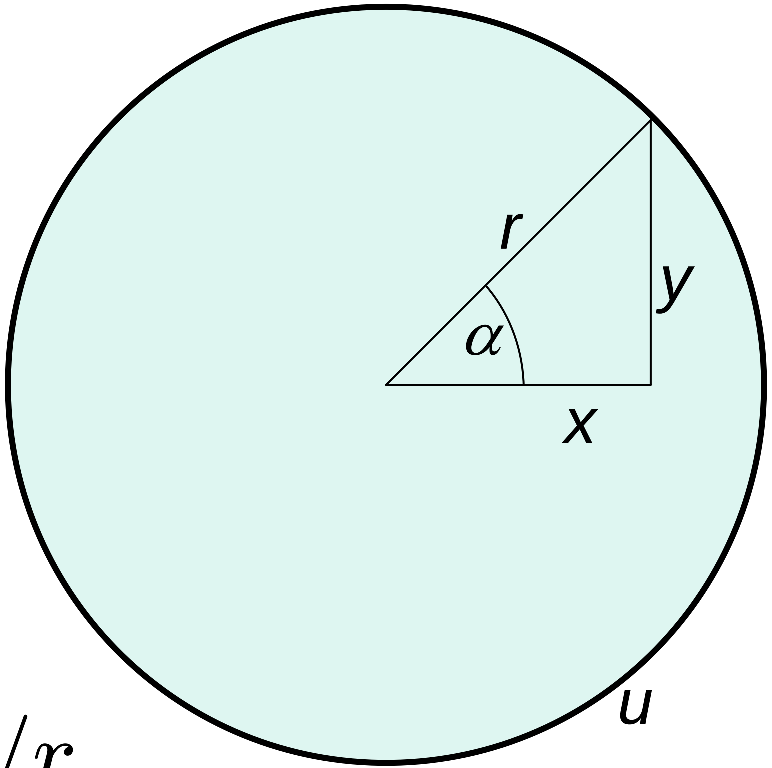
Kreis um (0,0), parametrisiert

$$x = r \cdot \cos(\alpha)$$

$$y = r \cdot \sin(\alpha)$$

$$u = 2 \cdot \pi \cdot r$$

$$step = \frac{2 \cdot \pi}{2 \cdot \pi \cdot r} = 1/r$$



TriCalcCircle

```
double step = 1.0/(double r);
double winkel;

for (winkel = 0.0;
     winkel < 2*Math.PI;
     winkel = winkel+step){

  setPixel((int) r*Math.sin(winkel)+0.5,
           (int) r*Math.cos(winkel)+0.5);
}
```

TriTableCircle

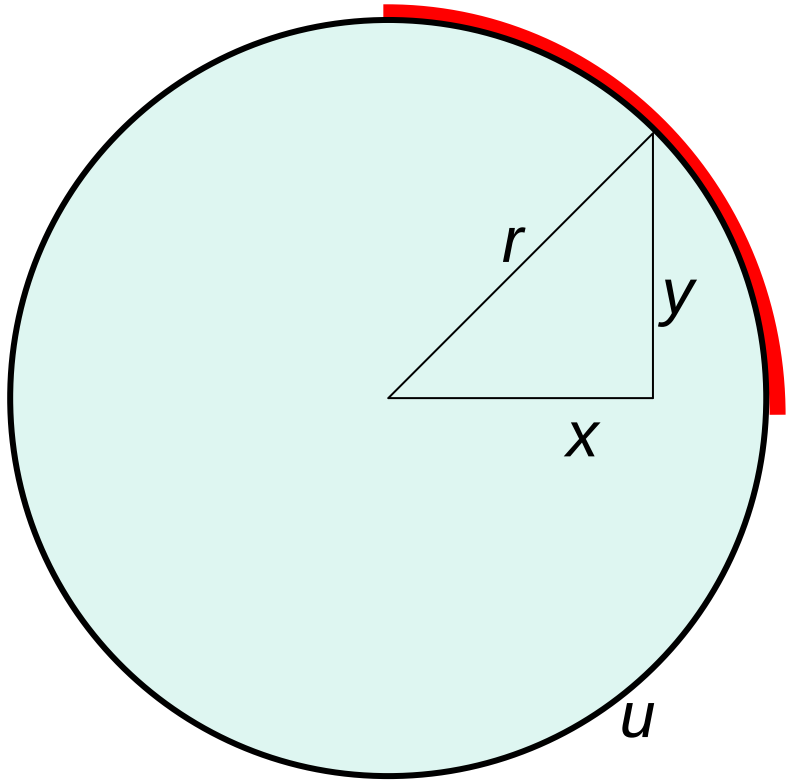
```
// Tabellen sin + cos seien berechnet
// für ganzzahlige Winkel von 0..360

int winkel;

for (winkel = 0;
     winkel < 360;
     winkel++){

  setPixel((int) r*sin[winkel] + 0.5,
           (int) r*cos[winkel] + 0.5);
}
```

Problem: konstante Zahl von Kreispunkten !

Kreis als Funktion im 1. Quadranden

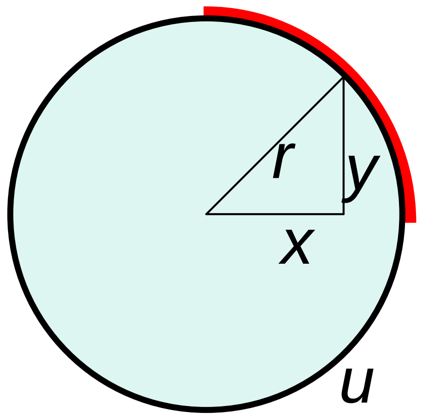
$$x^2 + y^2 = r^2$$

$$y = \sqrt{r^2 - x^2}$$



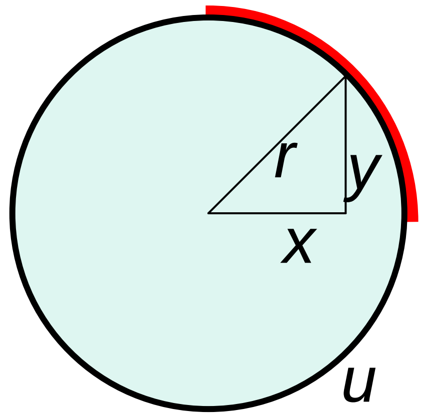
PythagorasCircle, die 1.

$$y = \sqrt{r^2 - x^2}$$




```
for (x=0; x <=r; x++){
  y = (int) Math.sqrt(r*r-x*x);
  setPixel(x,y);
}
```

PythagorasCircle, die 2.

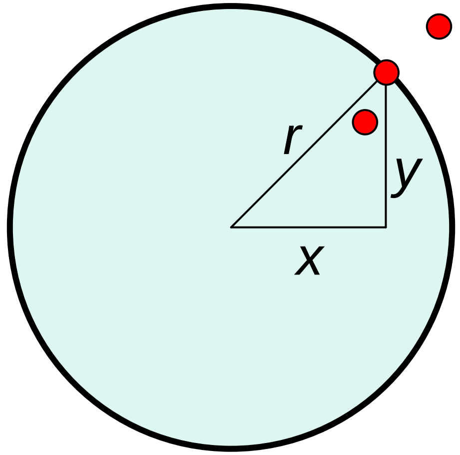


$$y = \sqrt{r^2 - x^2}$$

$$w = r^2,\ r^2 - 1,\ r^2 - 4,\ r^2 - 9,\ r^2 - 16, \ldots$$

```
d = 1;
w = r*r;
for (x=0; x <= r; x++) ){
  y = (int) Math.sqrt(w);
  setPixel(x,y);
  w = w-d
  d = d+2;
}
```

Punkt versus Kreis



$$x^2 + y^2 = r^2$$

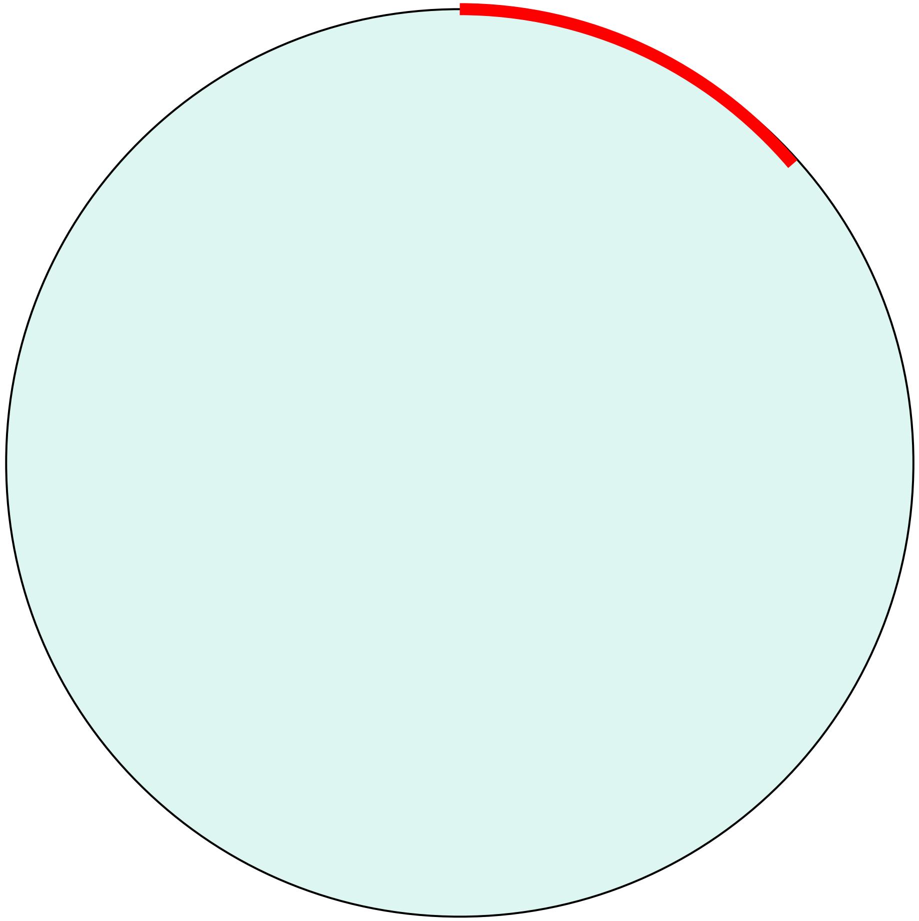
$$F(x,y) = x^2 + y^2 - r^2$$

$F(x,y)$ = 0 für (x,y) auf dem Kreis

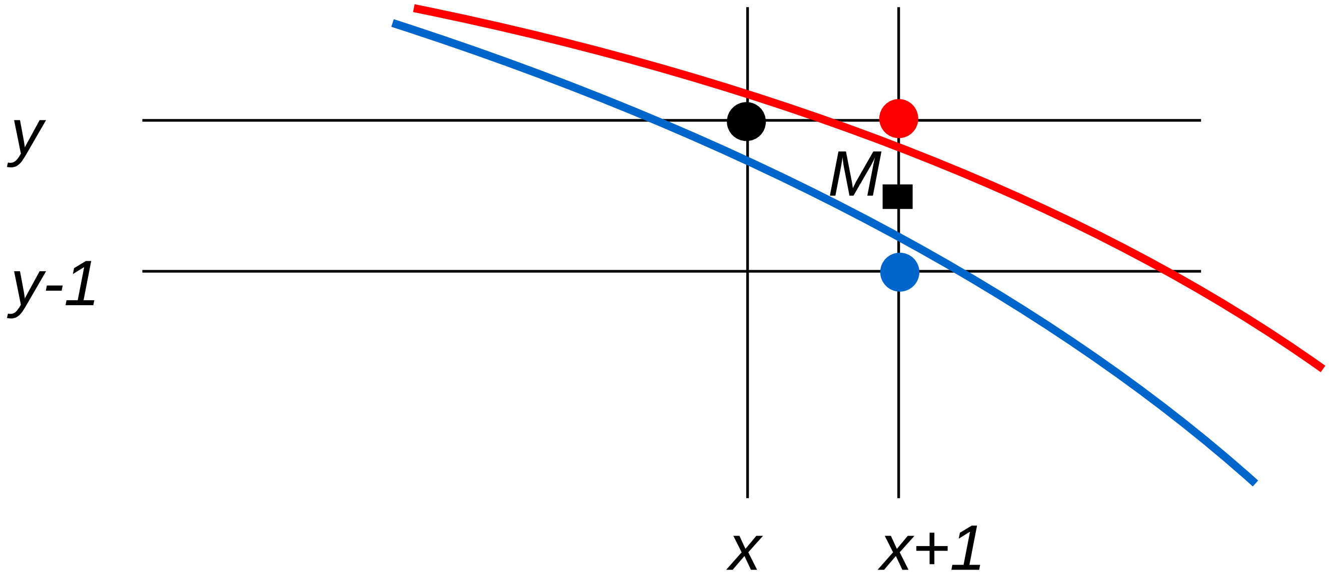
< 0 für (x,y) innerhalb des Kreises

> 0 für (x,y) außerhalb des Kreises

Kreis im 2. Oktanden



Entscheidungsvariable Δ



$\Delta = F(x+1, y-\frac{1}{2})$

$\Delta < 0 \Rightarrow M$ liegt innerhalb $\Rightarrow$ wähle $(x+1,y)$

$\Delta \geq 0 \Rightarrow M$ liegt außerhalb $\Rightarrow$ wähle $(x+1,y-1)$

Berechnung von Δ

$\Delta = F(x+1, y-½) = (x+1)^2+(y-½)^2-r^2$

$\Delta < 0 \Rightarrow$

$\Delta' = F(x+2, y-½) = (x+2)^2+(y-½)^2-r^2 =$

$\Delta+2x+3$

$\Delta \geq 0 \Rightarrow$

$\Delta' = F(x+2, y-3/2) = (x+2)^2+(y-3/2)^2-r^2 =$

$\Delta+2x-2y+5$

Startwert $\Delta = F(1, r-½) = 1^2+(r-½)^2 - r^2 =$

$5/4-r$

BresenhamCircle, die 1.

```
x = 0;
y = r;
delta = 5.0/4.0 - r;
while (y >= x) {
  setPixel(x,y);
  if (delta < 0.0) {
    delta = delta + 2*x + 3.0;
    x++;
  else {
    delta = delta + 2*x - 2*y + 5.0;
    x++;
    y--;
  }
}
```

Substitutionen

$d := delta - \frac{1}{4}$

$dx := 2x+3$

$dxy := 2x - 2y + 5$

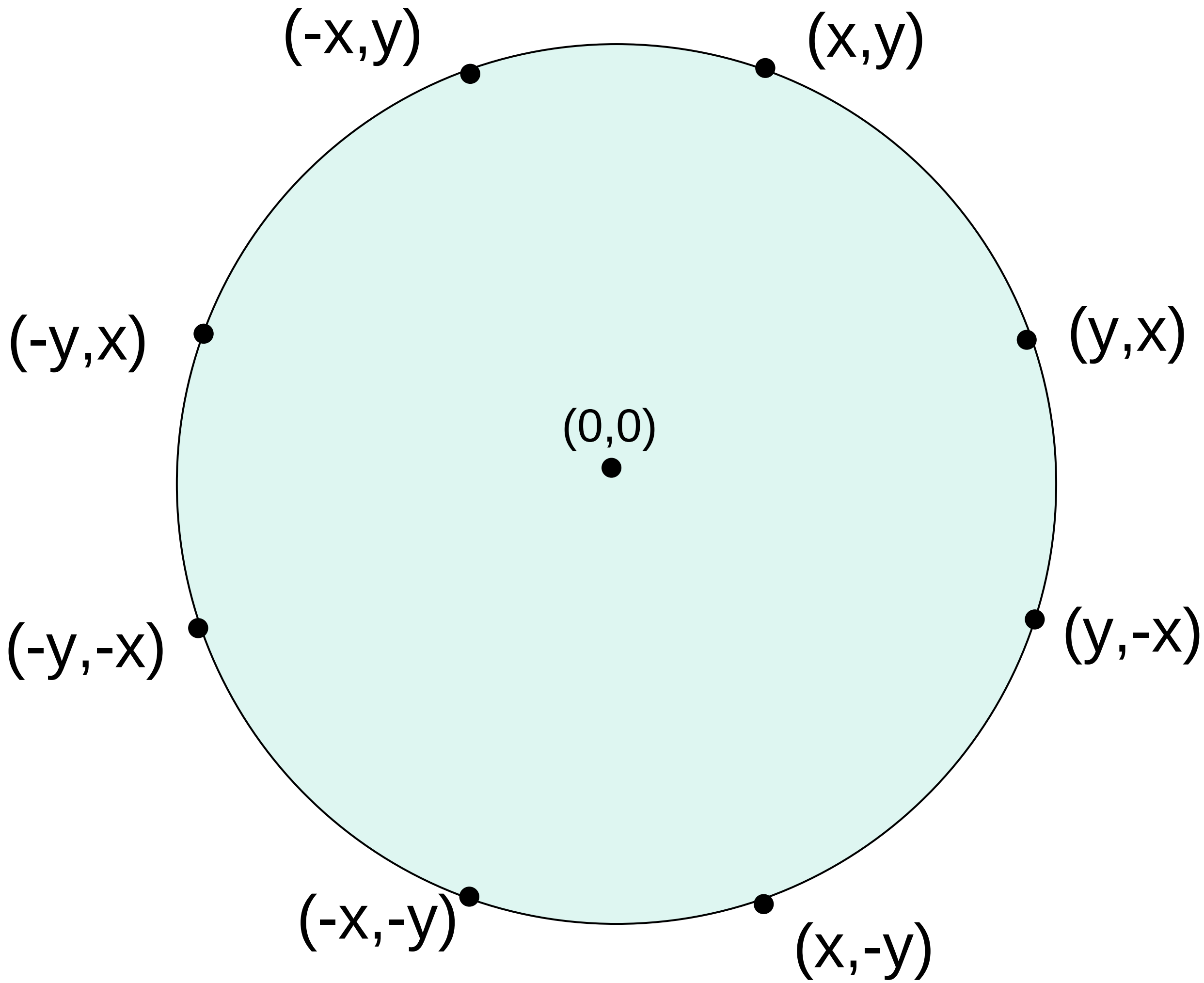
BresenhamCircle, die 2.

```
x = 0;
y = r;
~~delta = 5.0/4.0 - r~~;

while (y >= x) {
  setPixel(x,y);
  if (~~delta < 0.0~~) {
    ~~delta = delta + 2*x + 3.0~~;
    x++;
  else {
    ~~delta = delta+2*x-2*y+5.0~~;
    x++;
    y--;
  }
}
```

```
d = 1 - r;
dx = 3;
dxy = -2*r + 5;

(d < 0.0)

d = d + dx;
dx = dx + 2;
dxy = dxy + 2;

d   = d + dxy;
dx  = dx + 2;
dxy = dxy + 4;
```

d:=delta-1/4 dx:=2x+3 dxy:= 2x-2y+5

Oktanden-Symmetrie



BresenhamCircle, die 3.

```
x=0; y=r; d=1-r; x=3; dx=3; dxy=-2*r+5;


while (y>=x){

   setPixel(+x,+y);
   setPixel(+y,+x);
   setPixel(+y,-x);
   setPixel(+x,-y);
   setPixel(-x,-y);
   setPixel(-y,-x);
   setPixel(-y,+x);
   setPixel(-x,+y);

  if (d<0) {d=d+dx;  dx=dx+2; dxy=dxy+2; x++;}
  else     {d=d+dxy; dx=dx+2; dxy=dxy+4; x++; y--;}
}
```

BresenhamCircle

Zahl der erzeugten Punkte = $4 \cdot \sqrt{2} \cdot r$

= 10% unterhalb von $2 \cdot \pi \cdot r$

Kreis mit Radius r um Mittelpunkt (x,y):

~cg/2010/skript/Sources/drawBresenhamCircle.jav

Java-Applet:

~cg/2010/skript/Applets/2D-basic/App.html

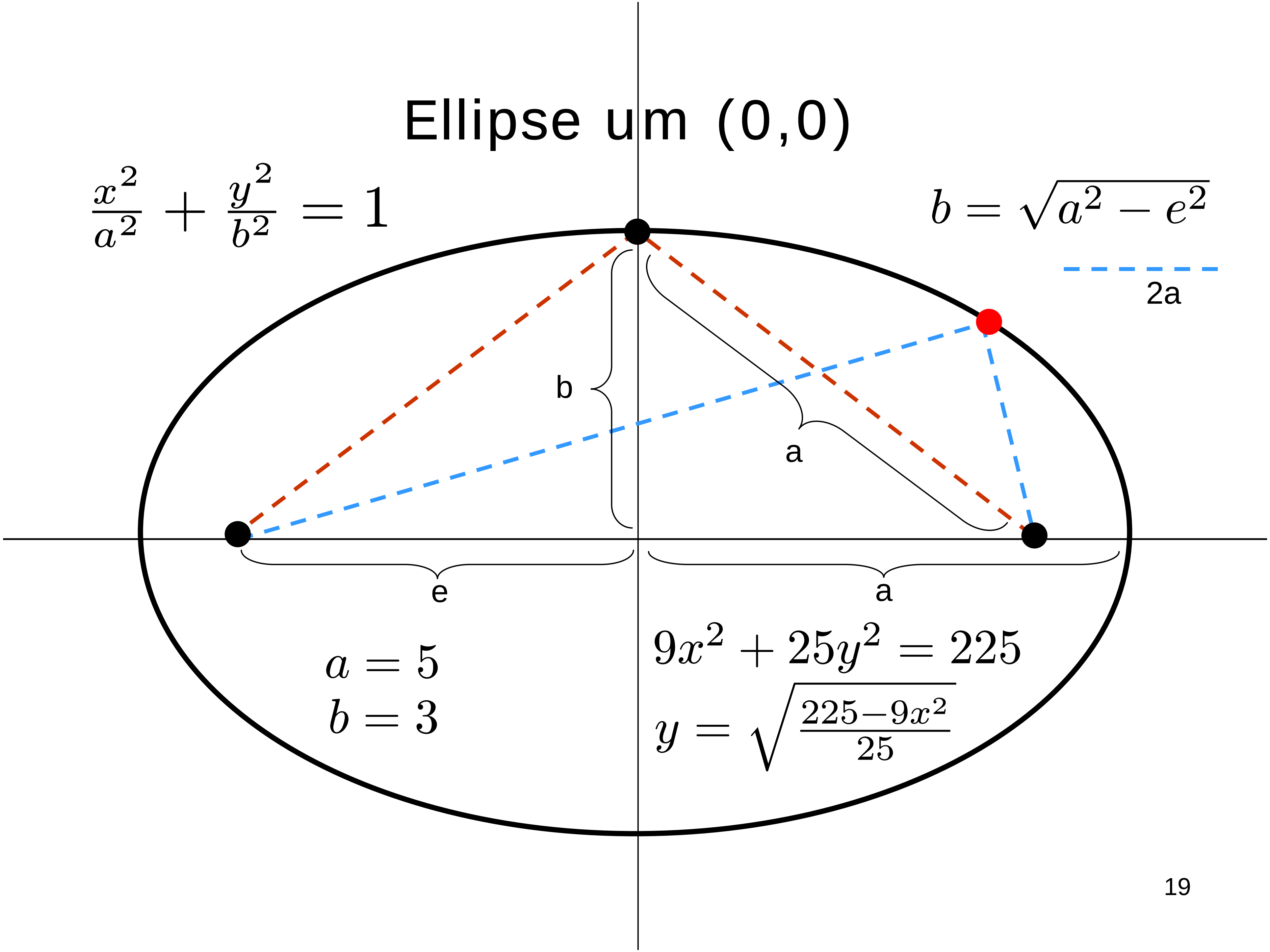
Java-Applet für Performance-Messung

~cg/2010/skript/Applets/circle/App.html

Ellipse um (0,0)

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

$$b = \sqrt{a^2 - e^2}$$

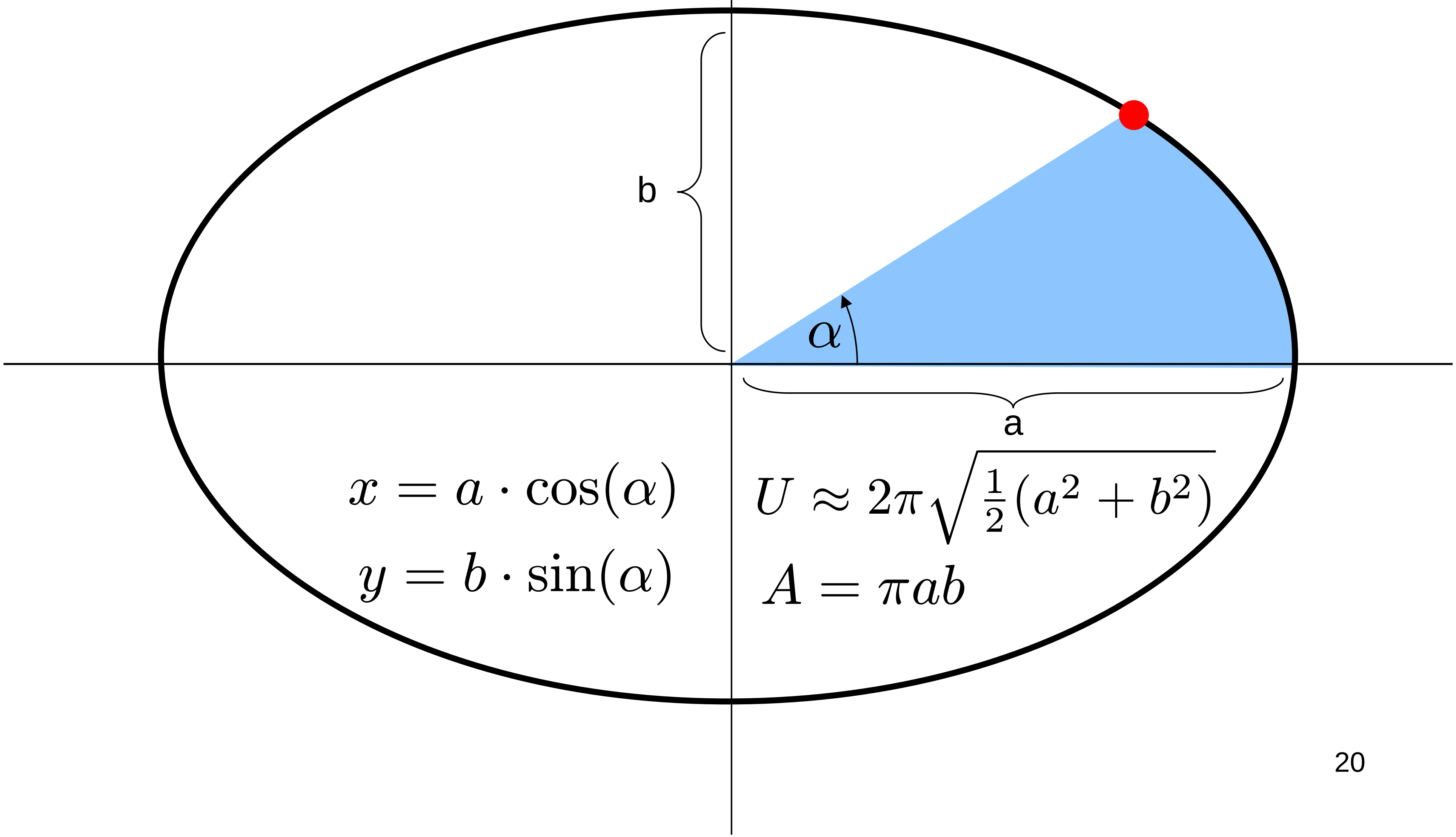


$$a = 5$$
$$b = 3$$

$$9x^2 + 25y^2 = 225$$

$$y = \sqrt{\frac{225 - 9x^2}{25}}$$

Ellipse um (0,0) parametrisiert



$$x = a \cdot \cos(\alpha)$$

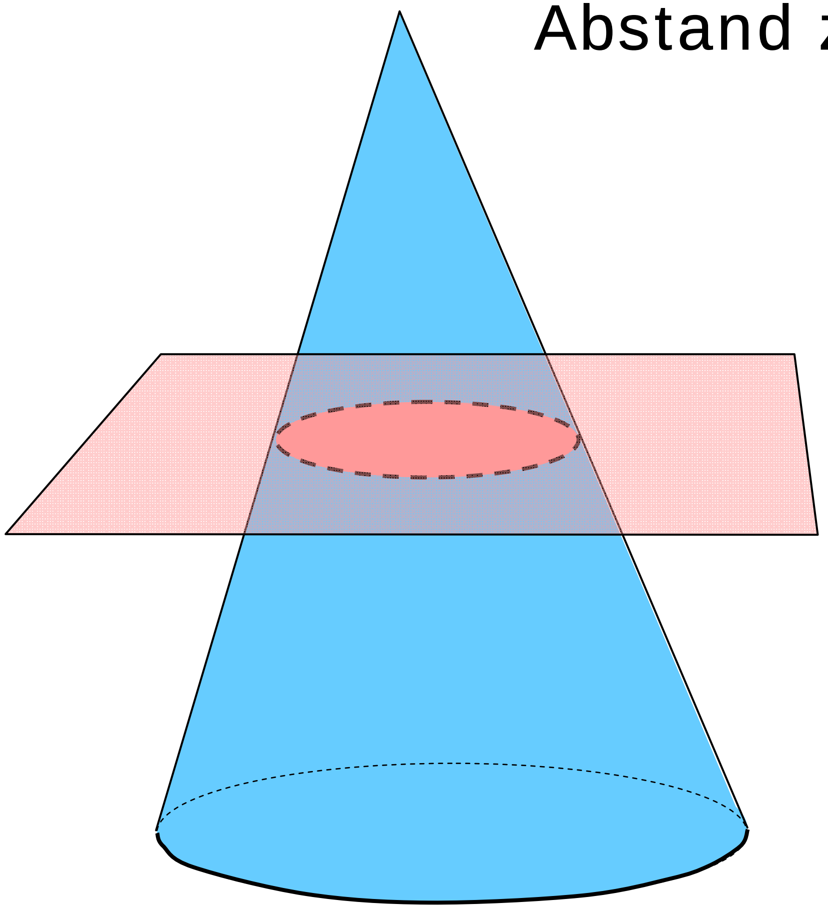
$$y = b \cdot \sin(\alpha)$$

$$U \approx 2\pi \sqrt{\tfrac{1}{2}(a^2 + b^2)}$$

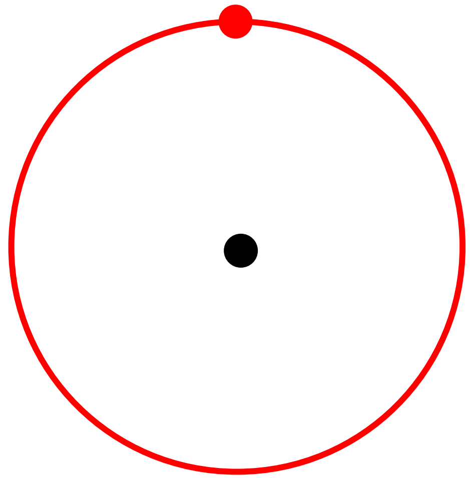
$$A = \pi a b$$

Kegelschnitt: Kreis

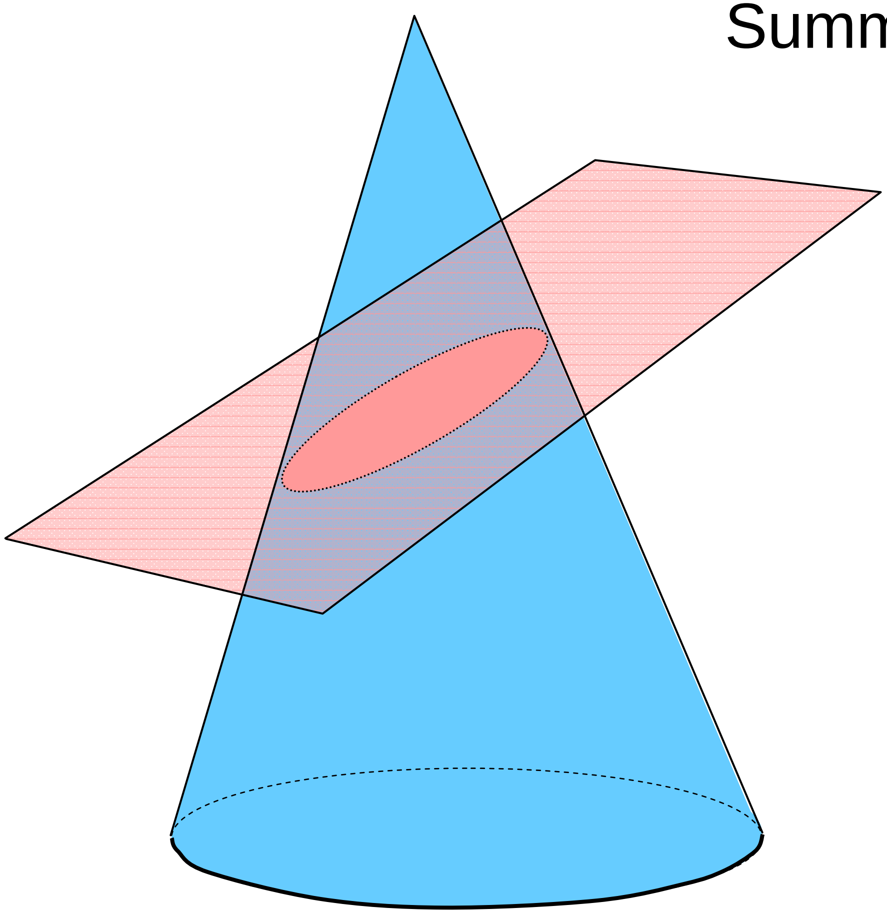
Abstand zu einem Punkt ist konstant



$$x^2 + y^2 = 1$$

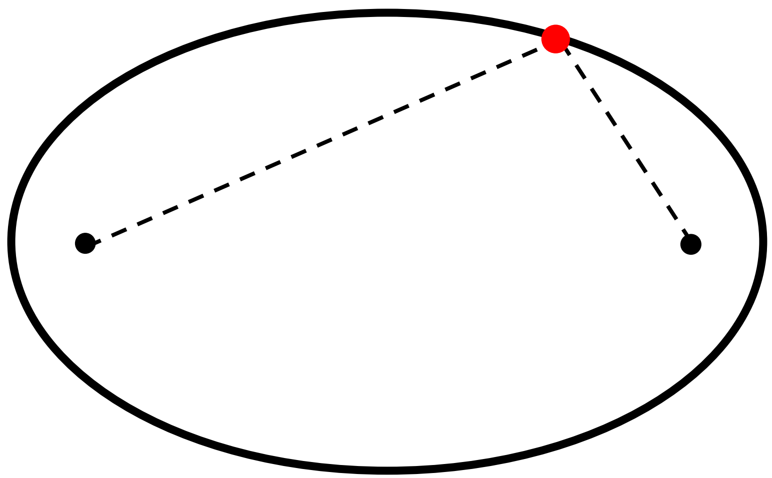


Kegelschnitt: Ellipse

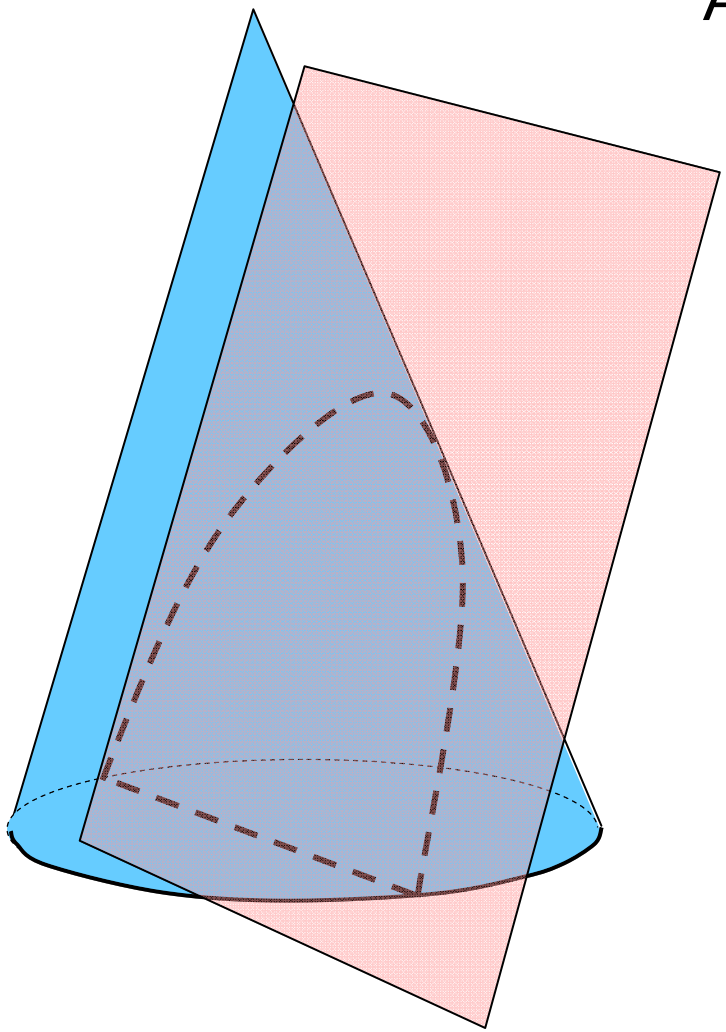


Summe der Abstände zu 2 Punkten ist konstant

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

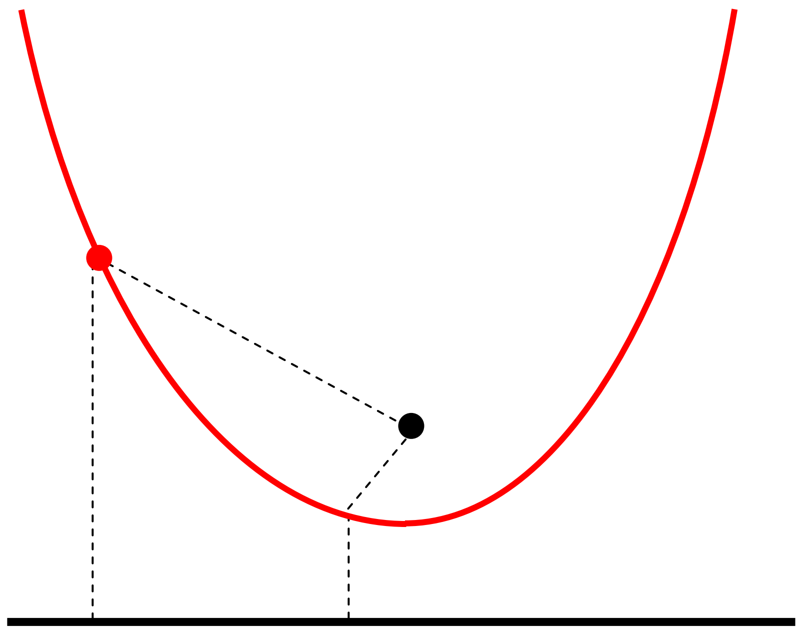


Kegelschnitt: Parabel



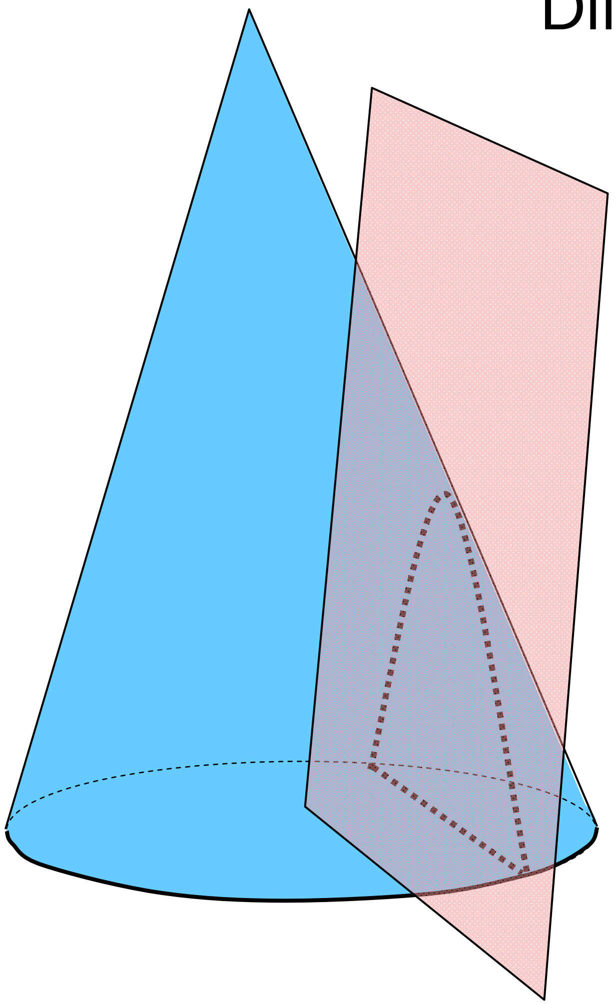
Abstand zu Punkt und Gerade
ist gleich

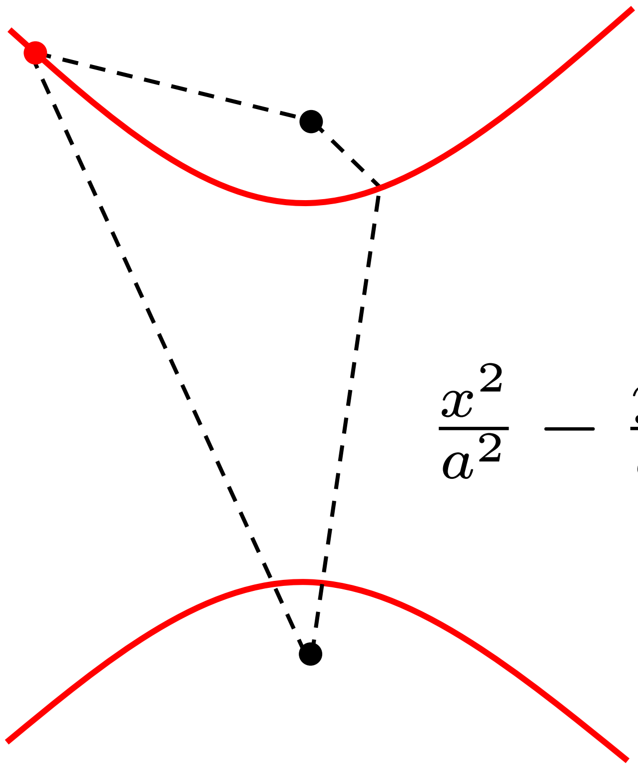
$$y = ax^2 + bx + c$$



Kegelschnitt: Hyperbel

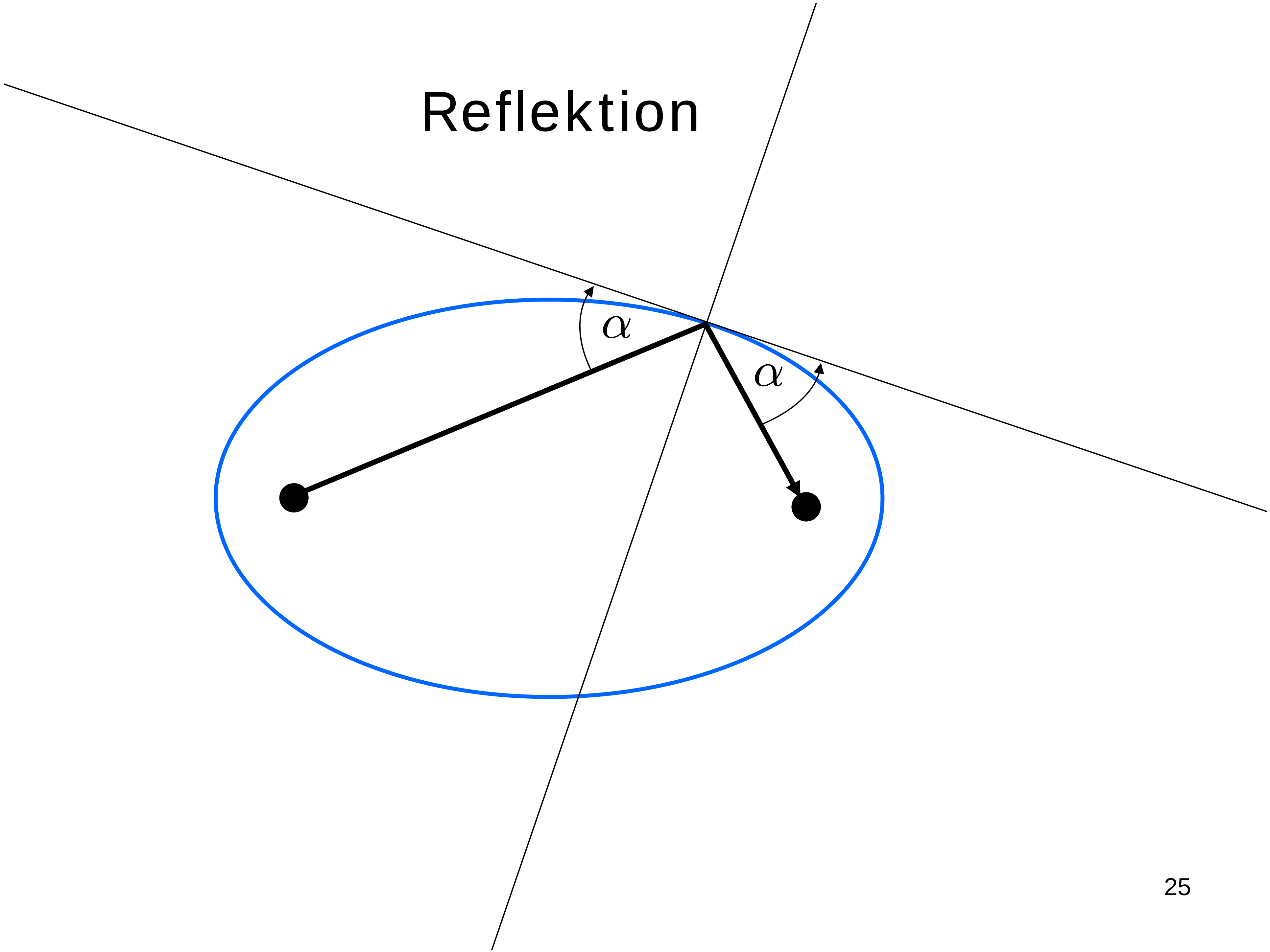
Differenz der Abstände zu 2 Punkten
ist konstant



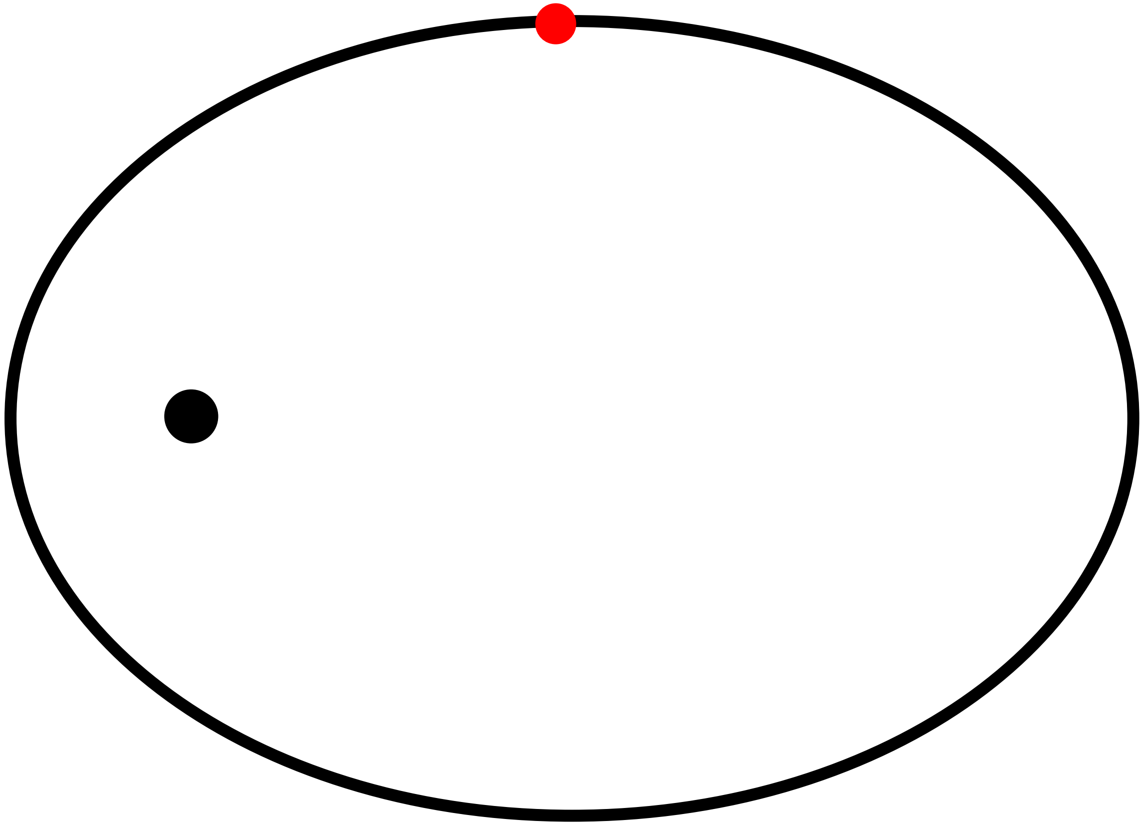


$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$$

Reflektion

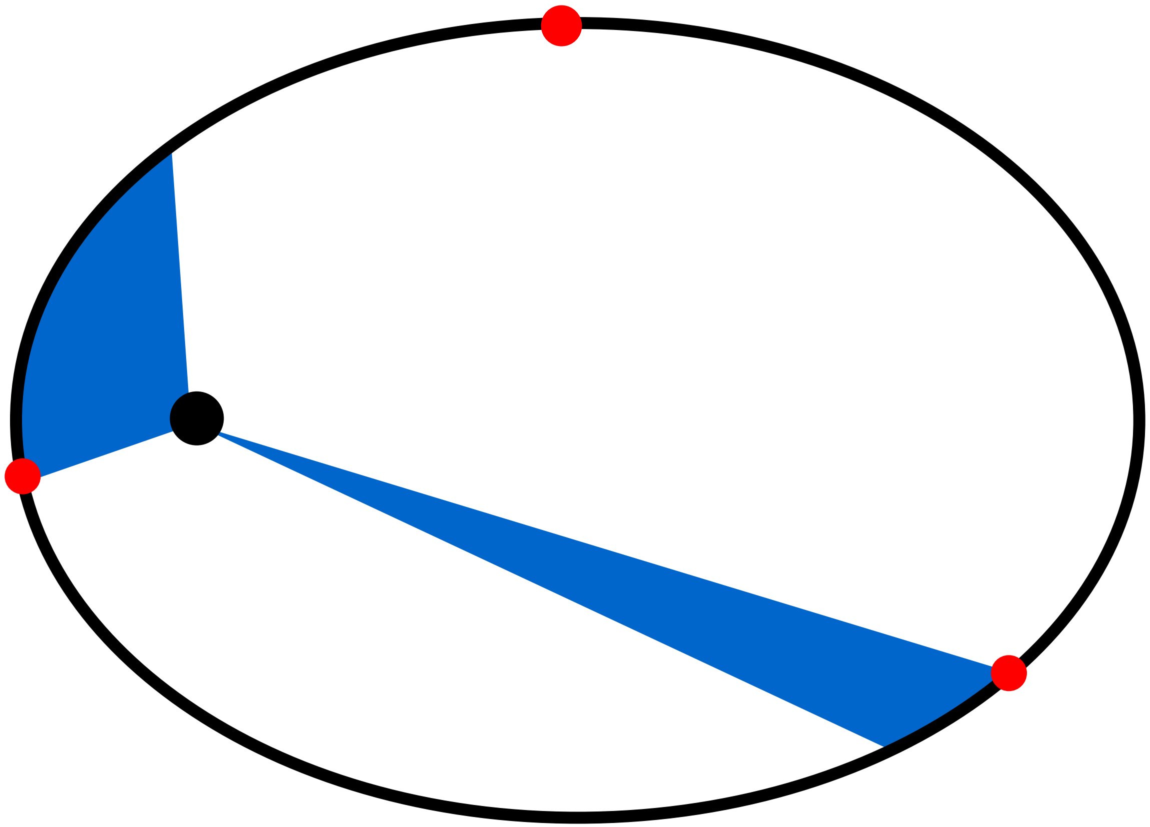


1. Keplersches Gesetz



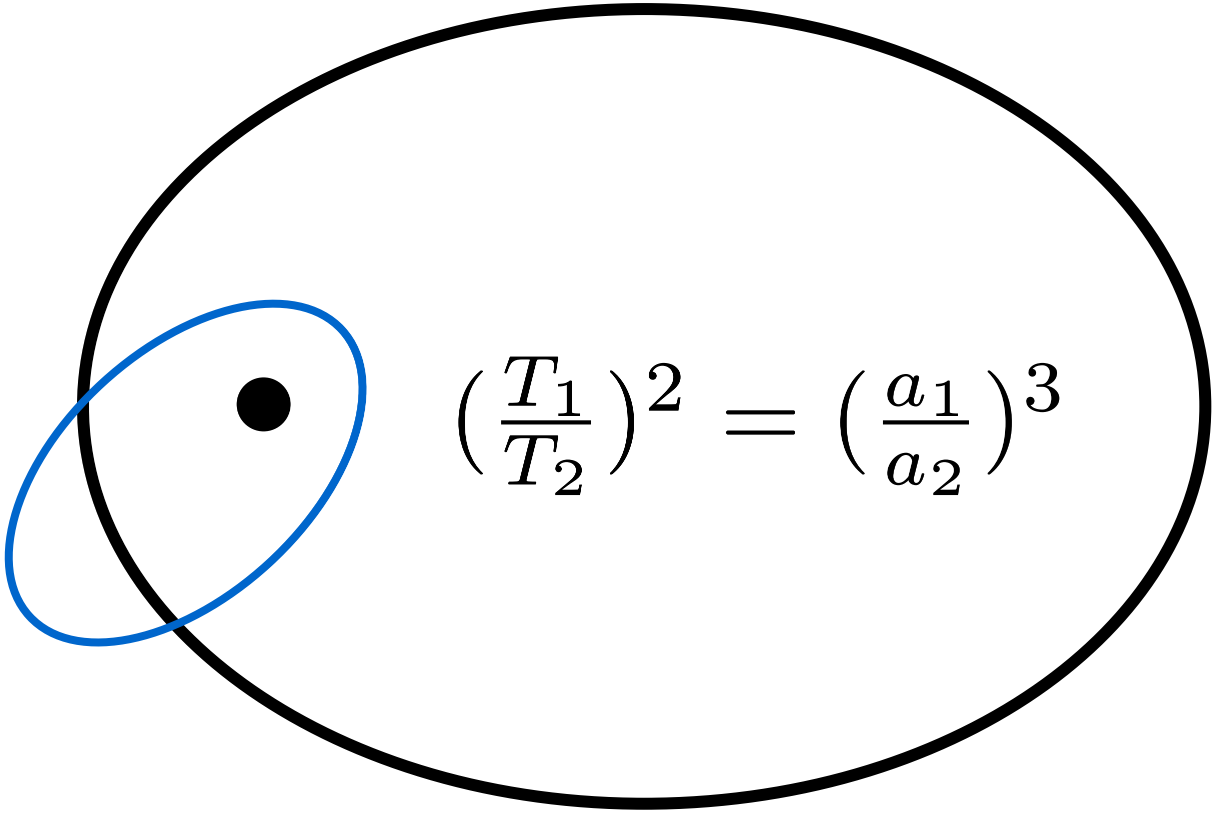
Die Planeten umkreisen die Sonne auf einer Ellipse

2. Keplersches Gesetz



In gleichen Zeiten überstreicht der Fahrstrahl gleiche Flächen

3. Keplersches Gesetz



$$\left(\frac{T_1}{T_2}\right)^2 = \left(\frac{a_1}{a_2}\right)^3$$

Die Quadrate der Umlaufzeiten verhalten sich
wie die Kuben der großen Halbachsen