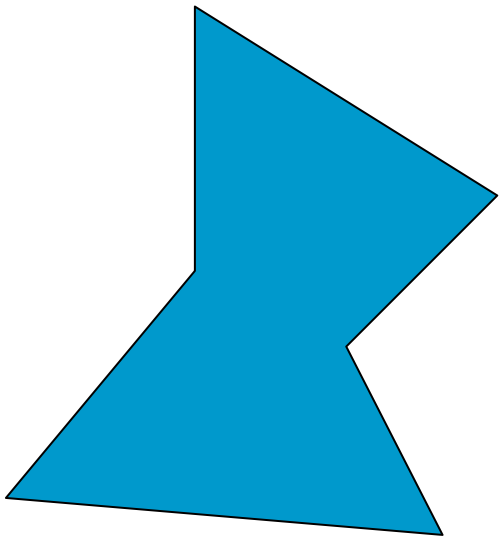
Computergrafik SS 2010

Oliver Vornberger

Kapitel 6:
2D-Transformationen

Vorlesung vom 26.04.10

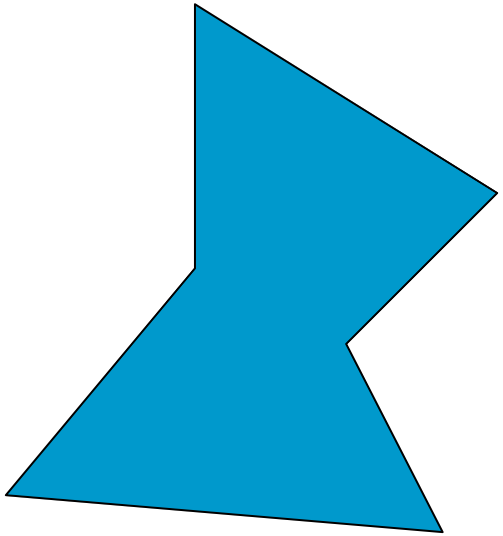
Translation



$$x := x + t_x$$

$$y := y + t_y$$

Skalierung



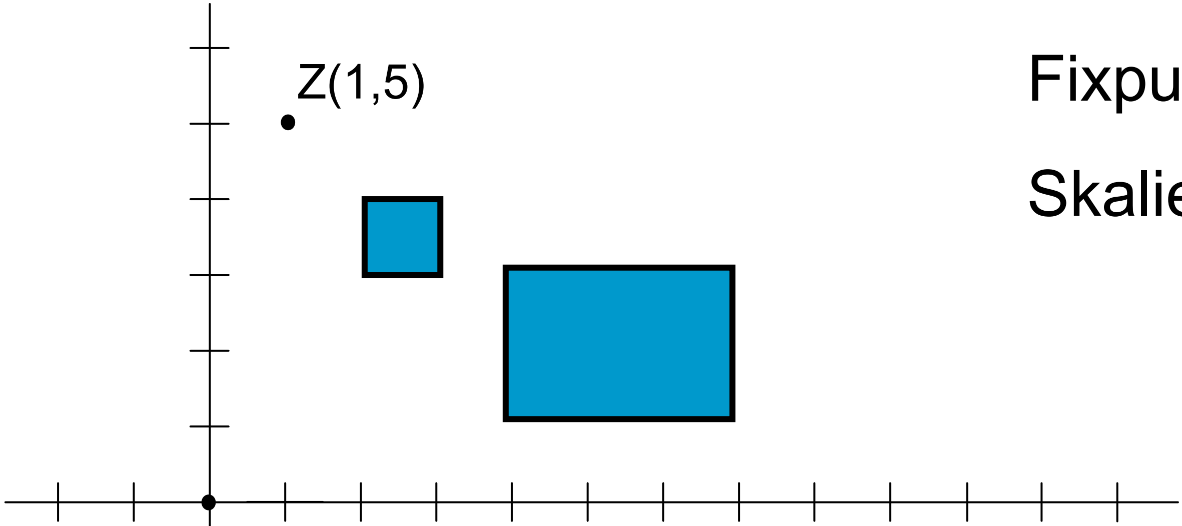
$$x := x \cdot s_x$$

$$y := y \cdot s_y$$

$s_x = s_y$ uniforme Skalierung

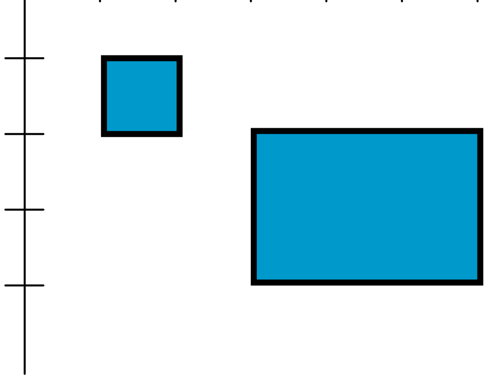
$s_x \neq s_y$ Verzerrung

Skalierung bzgl. Fixpunkt



Fixpunkt $Z_x=1$, $Z_y=5$

Skalierung $s_x=3$, $s_y=2$



Translation um $(-Z_x,-Z_y)$

Skalierung mit (s_x,s_y)

Translation um (Z_x,Z_y)

Skalierungsformel

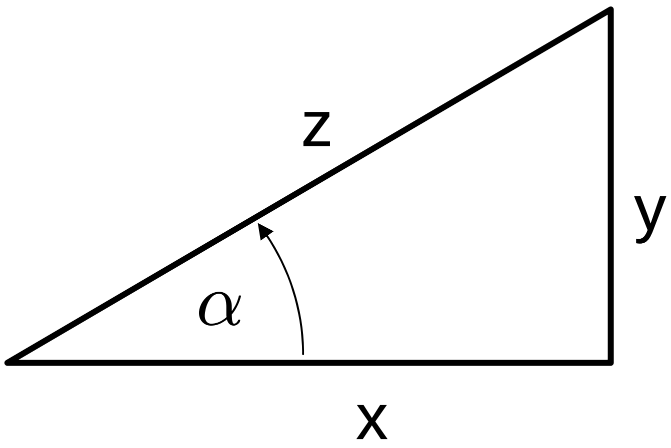
$$x' = (x - Z_x) \cdot s_x + Z_x$$

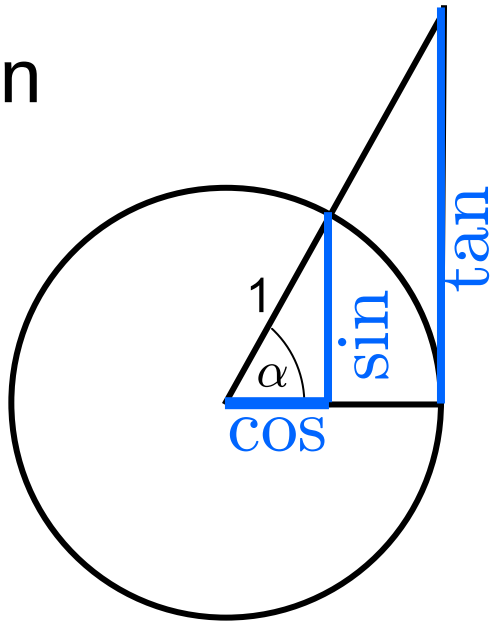
$$y' = (y - Z_y) \cdot s_y + Z_y$$

$$x' = x \cdot s_x - \underbrace{Z_x \cdot s_x + Z_x}_{d_x}$$

$$y' = y \cdot s_y - \underbrace{Z_y \cdot s_y + Z_y}_{d_y}$$

Trigonometrische Funktionen





$$\cos(\alpha) = \frac{x}{z} \qquad \sin(\alpha) = \frac{y}{z}$$

$$\tan(\alpha) = \frac{y}{x} \qquad \cot(\alpha) = \frac{x}{y}$$

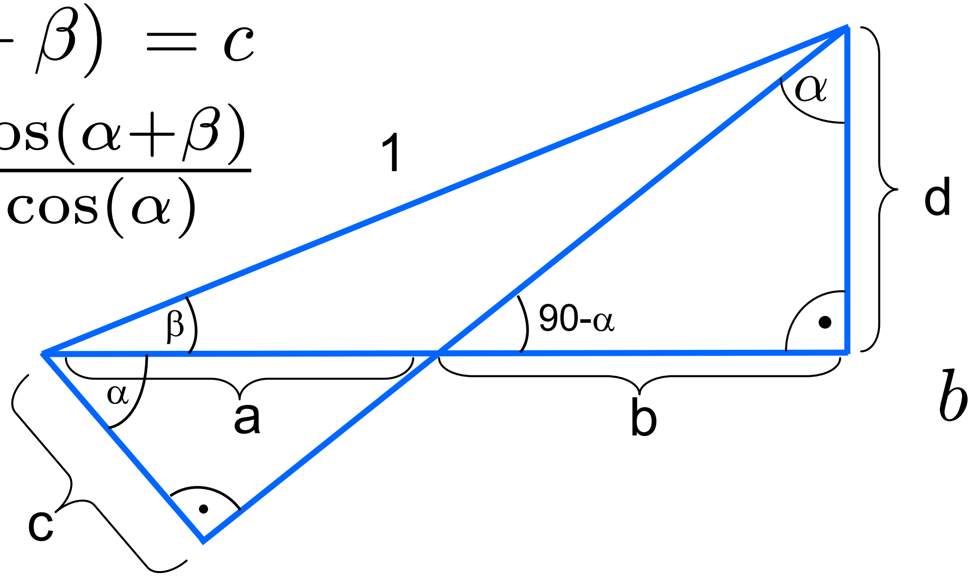
$$\tan(\alpha) = \frac{\sin(\alpha)}{\cos(\alpha)} \qquad \cot(\alpha) = \frac{1}{\tan(\alpha)}$$

Additionstheorem

$$\cos(\alpha) = c/a$$

$$\cos(\alpha+\beta) = c$$

$$a = \frac{\cos(\alpha+\beta)}{\cos(\alpha)}$$



$$\sin(\beta) = d$$

$$\tan(\alpha) = b/d$$

$$\tan(\alpha) = \frac{\sin(\alpha)}{\cos(\alpha)}$$

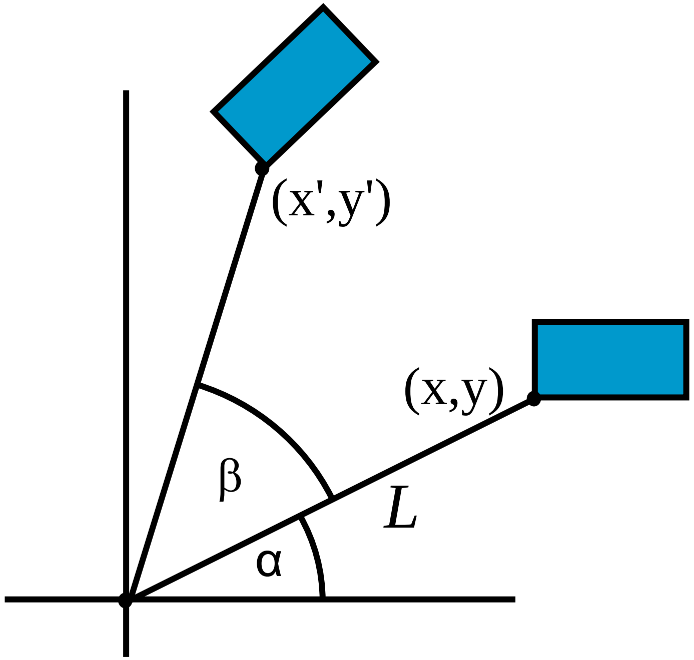
$$b = \frac{\sin(\alpha)}{\cos(\alpha)} \cdot \sin(\beta)$$

$$a + b = \cos(\beta) = \frac{cos(\alpha+\beta)}{cos(\alpha)} + sin(\beta) \cdot \frac{\sin(\alpha)}{cos(\alpha)}$$

$$cos(\alpha) \cdot \cos(\beta) = \cos(\alpha+\beta) + \sin(\alpha) \cdot \sin(\beta)$$

$$\cos(\alpha+\beta) = cos(\alpha) \cdot \cos(\beta) - \sin(\alpha) \cdot \sin(\beta)$$

Drehung

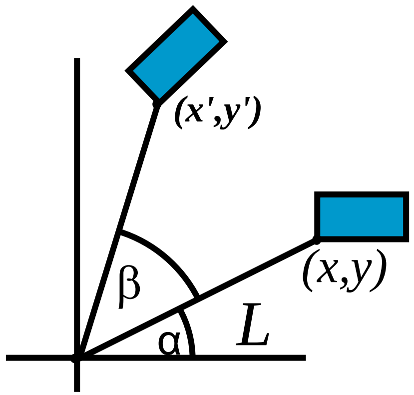


$$\cos(\alpha) = x/L$$

$$\sin(\alpha) = y/L$$

$$\cos(\alpha + \beta) = x'/L$$

$$\sin(\alpha + \beta) = y'/L$$



Formel für Drehung

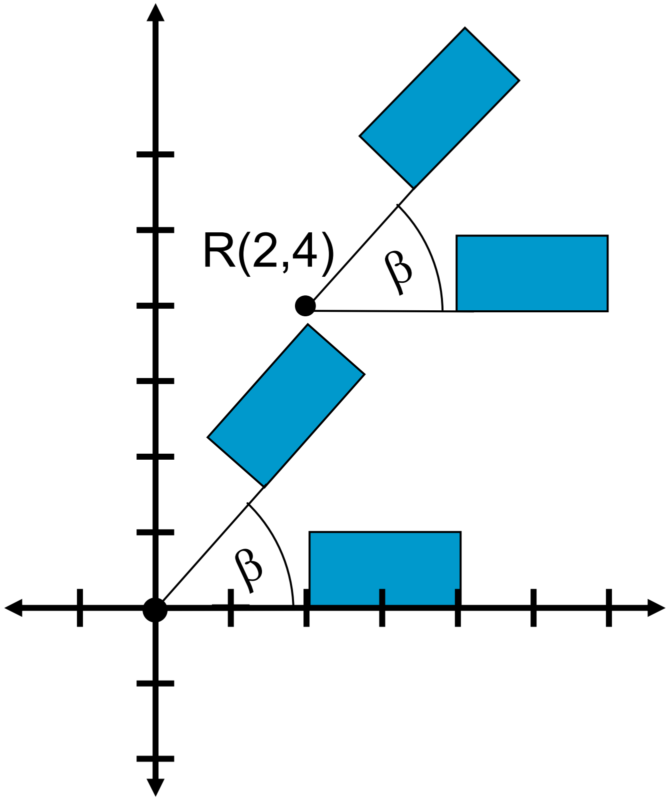
$$\cos(\alpha) = x/L \qquad \sin(\alpha) = y/L$$
$$\cos(\alpha+\beta) = x'/L \qquad \sin(\alpha+\beta) = y'/L$$

$$\sin(\alpha+\beta) = \cos(\beta)\cdot\sin(\alpha) + \sin(\beta)\cdot\cos(\alpha)$$
$$\cos(\alpha+\beta) = \cos(\beta)\cdot\cos(\alpha) - \sin(\beta)\cdot\sin(\alpha)$$

$$\cos(\alpha+\beta) = x'/L = \cos(\beta)\cdot\cos(\alpha) - \sin(\beta)\cdot\sin(\alpha)$$
$$x' = L\cdot\cos(\beta)\cdot x/L - \sin(\beta)\cdot y/L\cdot L$$
$$x' = x\cdot\cos(\beta) - y\cdot\sin(\beta)$$

$$\sin(\alpha+\beta) = y'/L = \cos(\beta)\cdot\sin(\alpha) + \sin(\beta)\cdot\cos(\alpha)$$
$$y' = L\cdot\cos(\beta)\cdot y/L + L\cdot\sin(\beta)x/L$$
$$y' = x\cdot\sin(\beta) + y\cdot\cos(\beta)$$

Rotation bzgl. Rotationszentrum



Translation um $(-R_x,-R_y)$

Rotation um β bzgl. (0,0)

Translation um (R_x,R_y)

Matrix für Rotation

$$x' := x \cdot \cos(\beta) - y \cdot \sin(\beta)$$

$$y' := x \cdot \sin(\beta) + y \cdot \cos(\beta)$$

$$\begin{pmatrix} x' \\ y' \end{pmatrix} := \begin{pmatrix} \cos(\beta) & -\sin(\beta) \\ \sin(\beta) & \cos(\beta) \end{pmatrix} \cdot \begin{pmatrix} x \\ y \end{pmatrix}$$

$$(x' \; y') := (x \; y) \cdot \begin{pmatrix} \cos(\beta) & \sin(\beta) \\ -\sin(\beta) & \cos(\beta) \end{pmatrix}$$

$$A \cdot B = (B^T \cdot A^T)^T$$

Matrix für Skalierung

$$x' := x \cdot s_x$$

$$y' := y \cdot s_y$$

$$\begin{pmatrix} x' \\ y' \end{pmatrix} := \begin{pmatrix} s_x & 0 \\ 0 & s_y \end{pmatrix} \cdot \begin{pmatrix} x \\ y \end{pmatrix}$$

Matrix für Translation

$$x' := x + t_x$$

$$y' := y + t_y$$

$$\begin{pmatrix} x' \\ y' \end{pmatrix} := \begin{pmatrix} ? & ? \\ ? & ? \end{pmatrix} \cdot \begin{pmatrix} x \\ y \end{pmatrix}$$

Homogene Koordinaten

$P = \begin{pmatrix} x \\ y \end{pmatrix}$ hat homogene Koordinaten $\begin{pmatrix} x \cdot w \\ y \cdot w \\ w \end{pmatrix}$ mit w $\neq$0

Zu den homogenen Koordinaten $\begin{pmatrix} x \\ y \\ w \end{pmatrix}$ gehört $P = \begin{pmatrix} x/w \\ y/w \end{pmatrix}$

Richtungsvektor $\vec{v} = \begin{pmatrix} x \\ y \end{pmatrix}$ hat homogene Koordinaten $\begin{pmatrix} x \\ y \\ 0 \end{pmatrix}$

Zum Punkt $\begin{pmatrix} 3 \\ 4 \end{pmatrix} \epsilon \ \mathbb{R}^2$ gehört die Ursprungsgerade $\begin{pmatrix} 3 \cdot w \\ 4 \cdot w \\ w \end{pmatrix} \epsilon \ \mathbb{R}^3$

Matrix für Translation

$$\begin{pmatrix} x' \\ y' \\ w' \end{pmatrix} := \begin{pmatrix} 1 & 0 & t_x \\ 0 & 1 & t_y \\ 0 & 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} x \\ y \\ 1 \end{pmatrix}$$

$$:= \begin{pmatrix} x + t_x \\ y + t_y \\ 1 \end{pmatrix}$$

Beispiel für Translation

$$\begin{pmatrix} 7 \\ 8 \\ 1 \end{pmatrix} := \begin{pmatrix} 1 & 0 & 2 \\ 0 & 1 & 2 \\ 0 & 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} 5 \\ 6 \\ 1 \end{pmatrix}$$

$$\begin{pmatrix} 21 \\ 24 \\ 3 \end{pmatrix} := \begin{pmatrix} 1 & 0 & 2 \\ 0 & 1 & 2 \\ 0 & 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} 15 \\ 18 \\ 3 \end{pmatrix}$$

Matrix für Skalierung

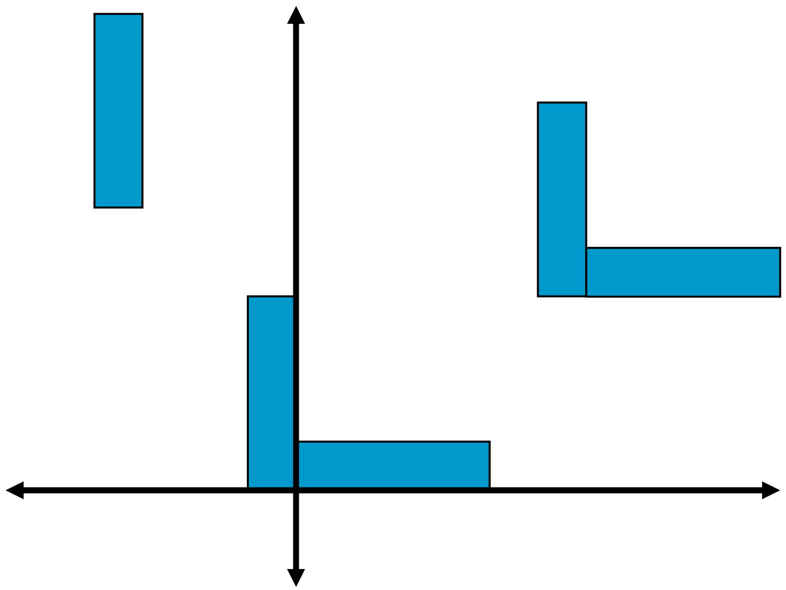
$$\begin{pmatrix} x' \\ y' \\ w' \end{pmatrix} := \begin{pmatrix} s_x & 0 & 0 \\ 0 & s_y & 0 \\ 0 & 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} x \\ y \\ 1 \end{pmatrix}$$

Matrix für Rotation

$$\begin{pmatrix} x' \\ y' \\ w' \end{pmatrix} := \begin{pmatrix} \cos(\beta) & -\sin(\beta) & 0 \\ \sin(\beta) & \cos(\beta) & 0 \\ 0 & 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} x \\ y \\ 1 \end{pmatrix}$$

Verknüpfung von Transformationen

- assoziativ: A•B•C = (A•B)•C = A•(B •C)
- nicht kommunativ: A • B ≠ B • A
- Drehung um 90° + Verschieben um (4,3) ≠ Verschieben um (4,3) + Drehung um 90°



Rotation bzgl (3,5) um 60°

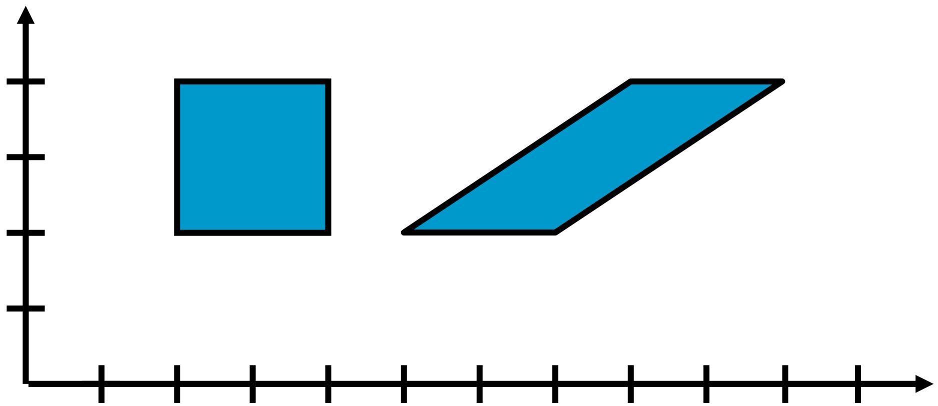
$$A = \begin{pmatrix} 1 & 0 & -3 \\ 0 & 1 & -5 \\ 0 & 0 & 1 \end{pmatrix} \qquad C = \begin{pmatrix} 1 & 0 & 3 \\ 0 & 1 & 5 \\ 0 & 0 & 1 \end{pmatrix}$$

$$B = \begin{pmatrix} 0.5000000 & -0.8660254 & 0.000000 \\ 0.8660254 & 0.5000000 & 0.000000 \\ 0.0000000 & 0.0000000 & 1.000000 \end{pmatrix}$$

$$D = C \cdot B \cdot A \ =$$

$$\begin{pmatrix} 0.5000000 & -0.8660254 & 2.8301270 \\ 0.8660254 & 0.5000000 & -0.0980762 \\ 0.0000000 & 0.0000000 & 1.0000000 \end{pmatrix}$$

Matrix für Scherung in x-Richtung

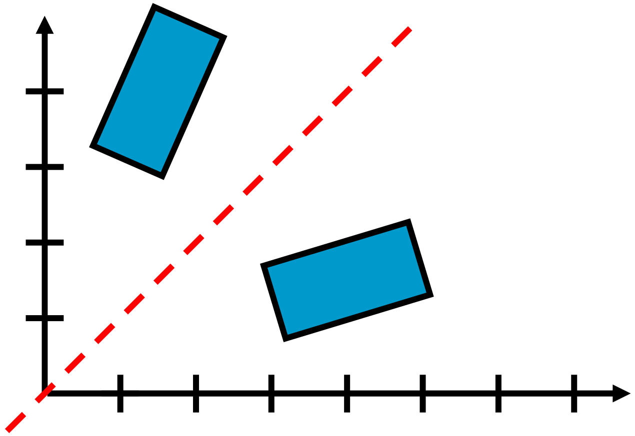


$$x' := x + m \cdot y$$

$$y' := y$$

$$\begin{pmatrix} x' \\ y' \\ w' \end{pmatrix} := \begin{pmatrix} 1 & m & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} x \\ y \\ 1 \end{pmatrix}$$

Matrix für Spiegelung an Hauptdiagonale

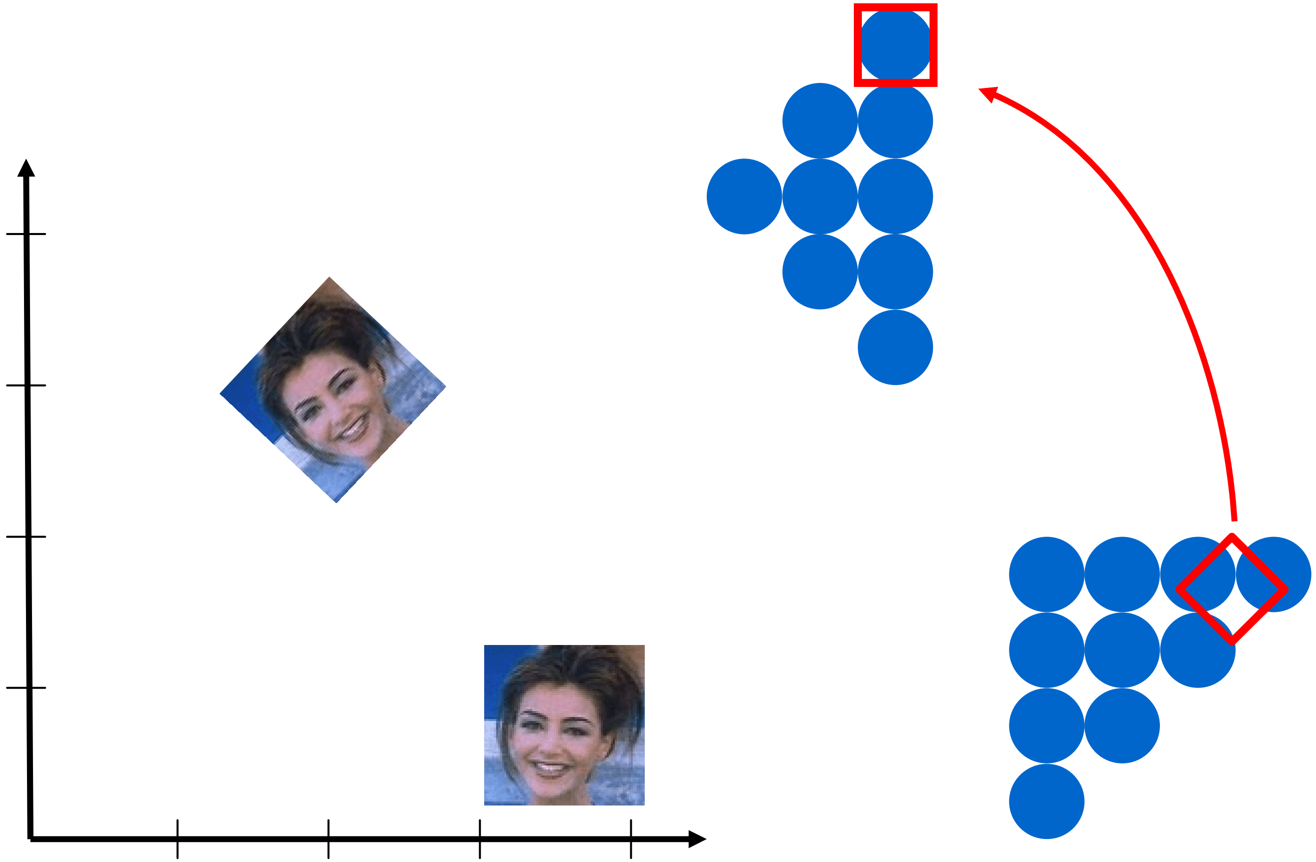


$$x' := y$$

$$y' := x$$

$$\begin{pmatrix} x' \\ y' \\ w' \end{pmatrix} := \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} x \\ y \\ 1 \end{pmatrix}$$

Pixel-Transformation



Transformations-Implementation

Java-Applet:

~cg/2010/skript/Applets/2D-trafo/App.html

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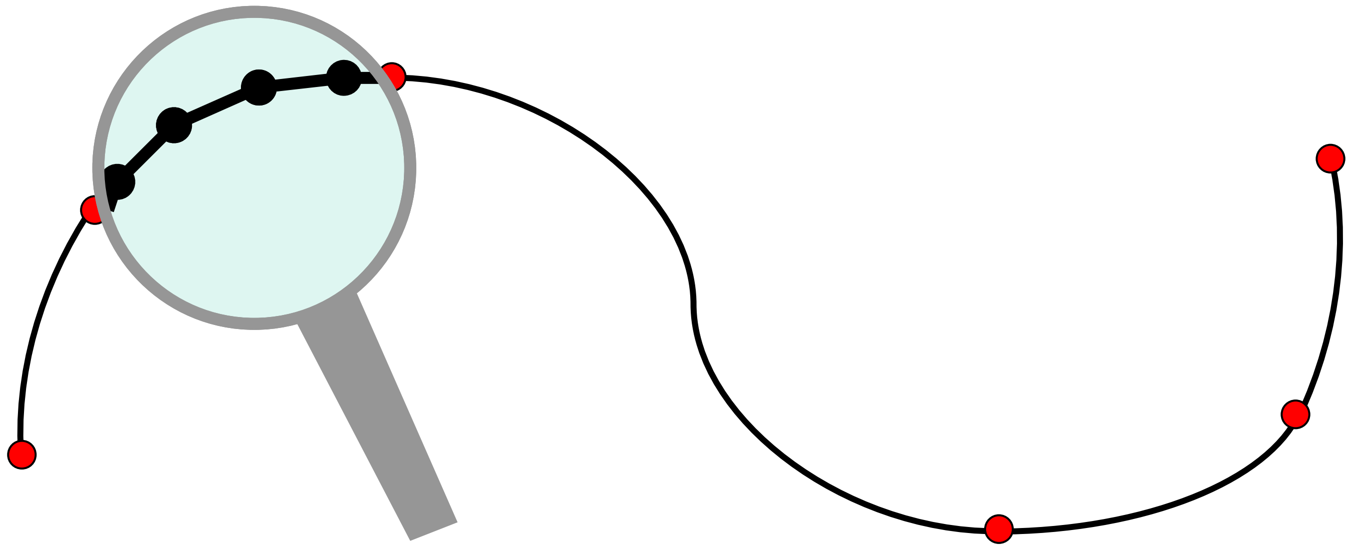
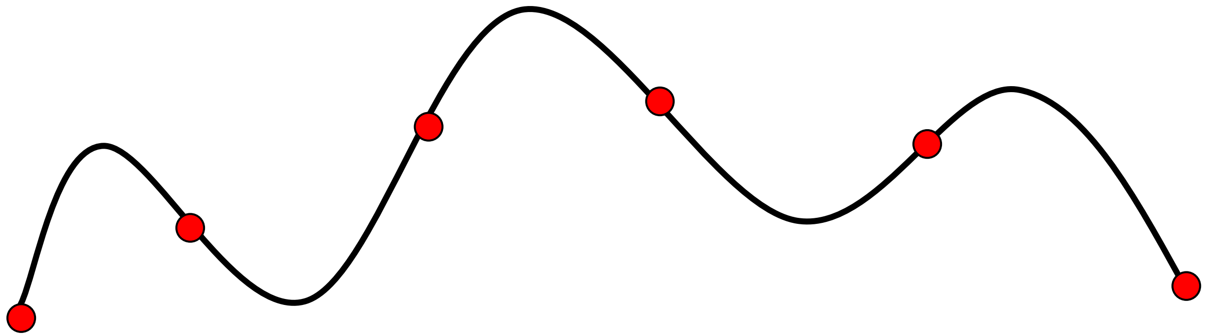
Kapitel 7:
2D-Kurven

Spezifikation einer Kurve



Stützpunkte $P_0, P_1, \ldots, P_n$

Algebraischer Ansatz

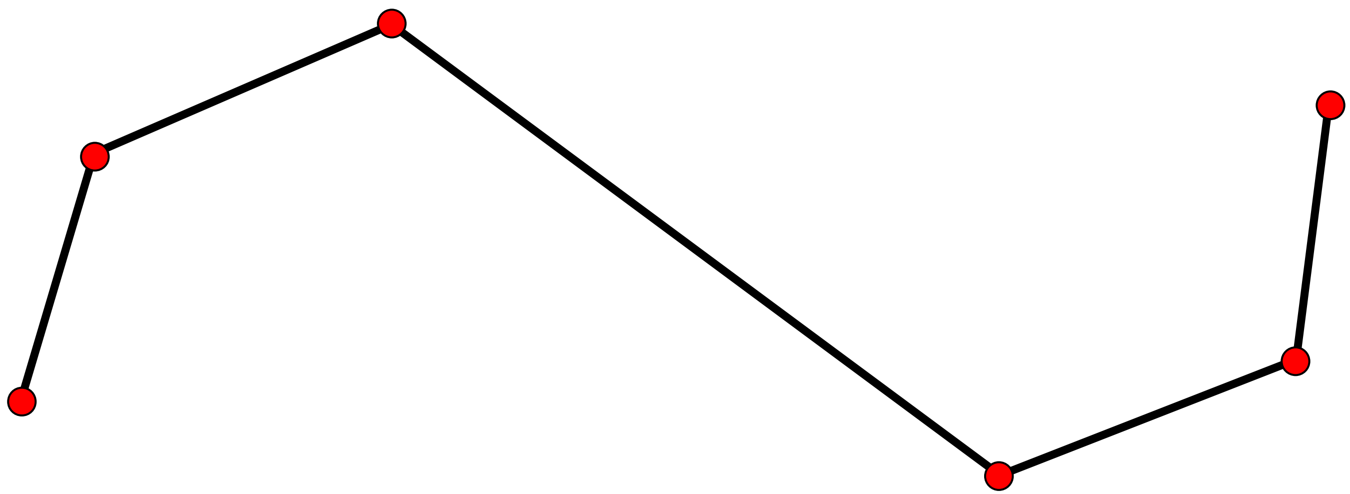


Bestimme n+1 Koeffizienten für Polynom n-ten Grades

$$y = a_n \cdot x^n + a_{n-1} \cdot x^{n-1} + ... + a_1 \cdot x + a_0$$

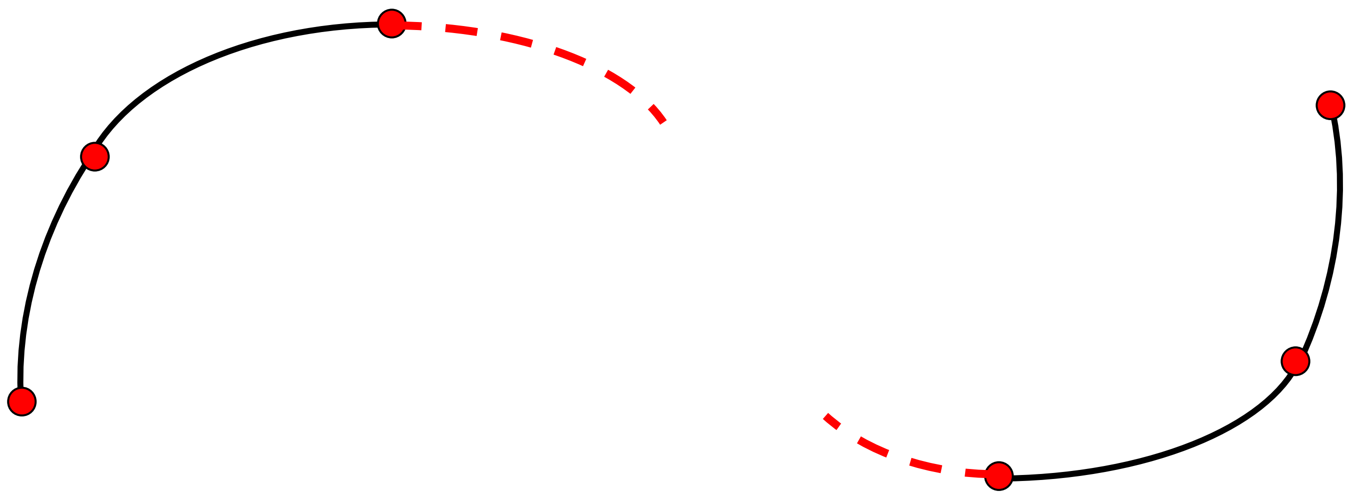
Oszillation! Rechenaufwand! Rundungsfehler !

lineare Splines



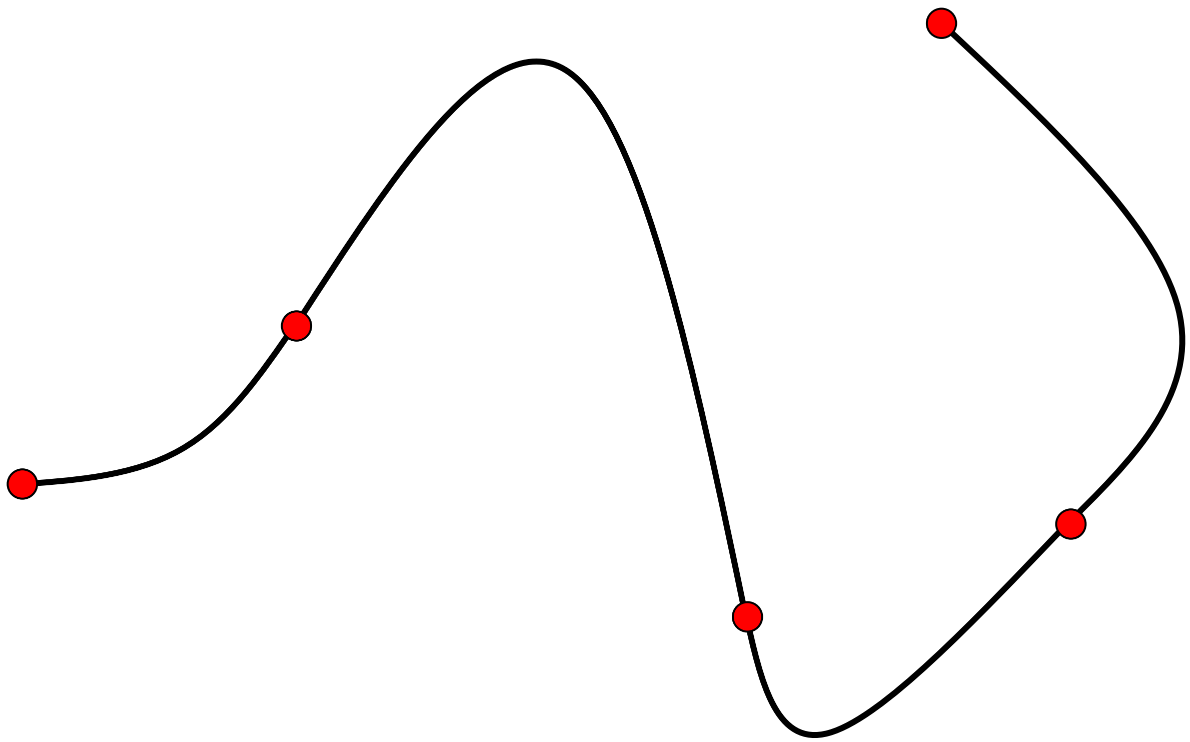
verbinde zwei aufeinanderfolgende Punkte
durch eine Gerade

quadratische Splines

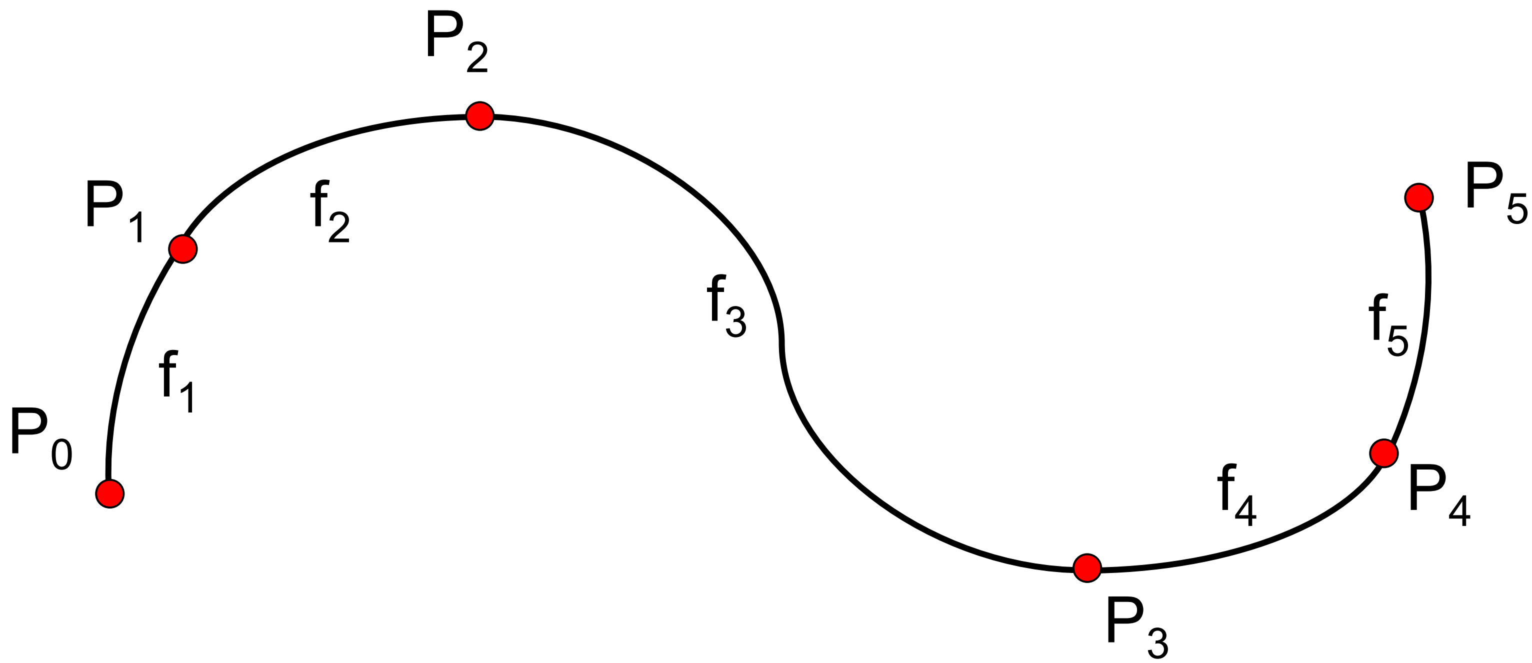


verbinde zwei aufeinanderfolgende Punkte
durch eine Kurve 2. Grades

quadratische Splines

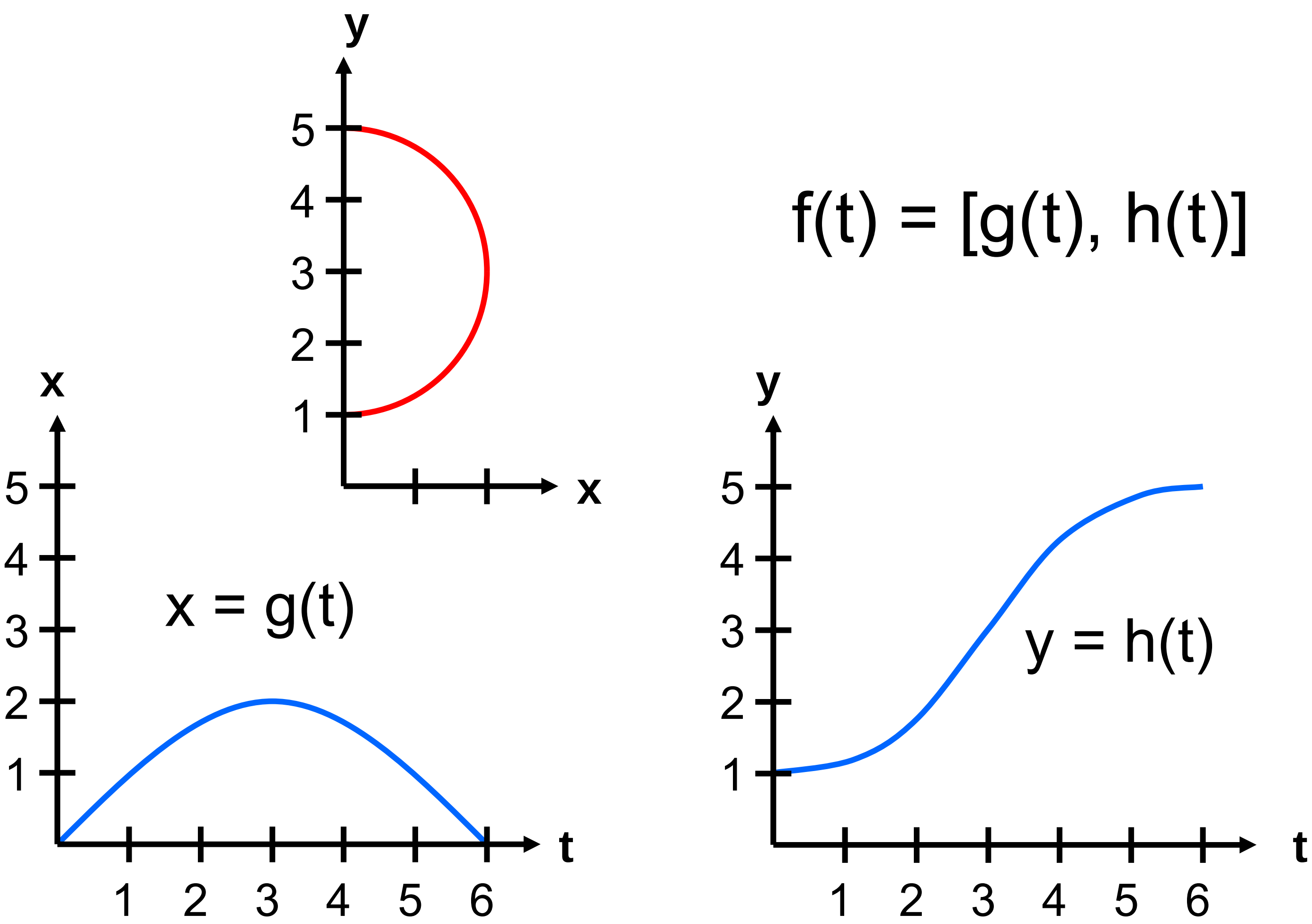


kubische Splines

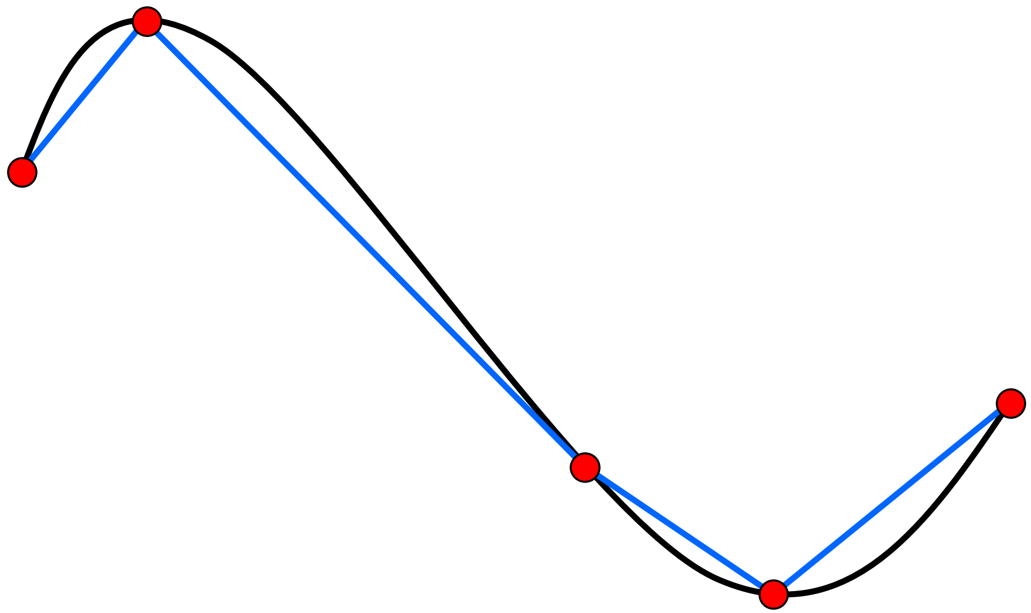


Verbinde zwei aufeinanderfolgende Punkte durch eine Kurve 3. Grades

Parametrisierte Kurvengleichung



Intervallgrenzen



$$\Delta_i = \sqrt{(x_i - x_{i+1})^2 + (y_i - y_{i+1})^2}$$

Kurvenabschnitte

Gesucht ist pro Intervall i, i=1, …, n

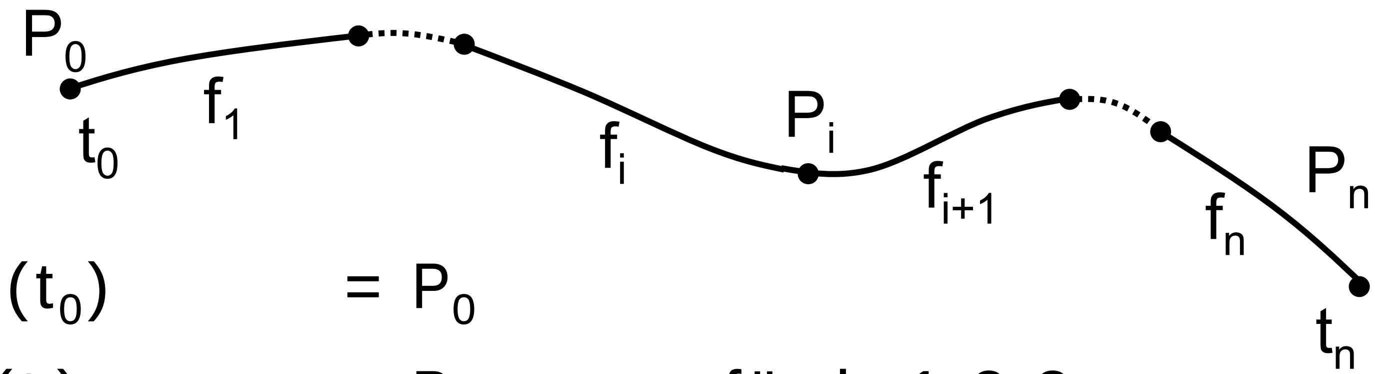
$$f_i(t) = a_i + b_i \cdot t + c_i \cdot t^2 + d_i \cdot t^3$$

genauer:

$$g_i(t) = a^g_i + b^g_i \cdot t + c^g_i \cdot t^2 + d^g_i \cdot t^3$$

$$h_i(t) = a^h_i + b^h_i \cdot t + c^h_i \cdot t^2 + d^h_i \cdot t^3$$

Gleichungssystem



$$
\begin{aligned}
f_1(t_0) &= P_0 && \\
f_i(t_i) &= P_i && \text{für } i=1,2,3,\ldots,n \\
f_i(t_i) &= f_{i+1}(t_i) && \text{für } i=1,2,3,\ldots,n-1 \\
f'_i(t_i) &= f'_{i+1}(t_i) && \text{für } i=1,2,3,\ldots,n-1 \\
f''_i(t_i) &= f''_{i+1}(t_i) && \text{für } i=1,2,3,\ldots,n-1 \\
f_1''(t_0) &= 0 && \\
f_n''(t_n) &= 0 &&
\end{aligned}
$$

$2 \cdot 4n$ Gleichungen mit
$2 \cdot 4n$ Unbekannten

Approximation

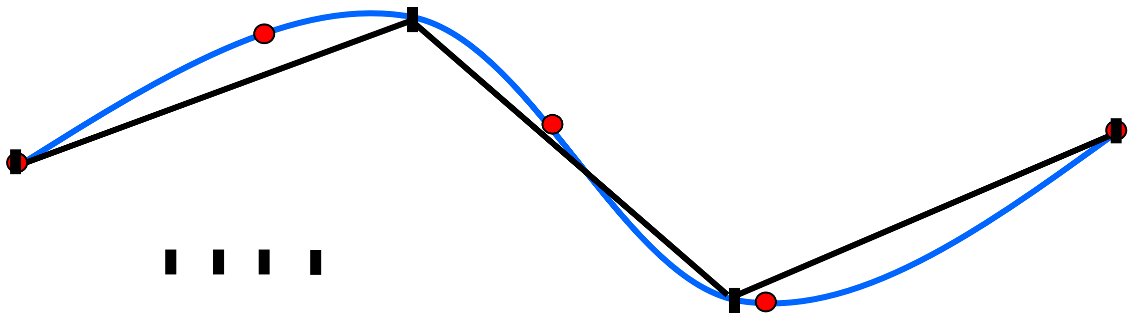
Gegeben n+1 Stützpunkte $P_0, P_1, \ldots, P_n$

Berechne Kurvenabschnitte $f_1, f_2, \ldots, f_n$

Bestimme Zahl der Interpolationspunkte k

Verteile längs der Kurvenabschnitte

Zeichne k-1 Geradenabschnitte



Java-Applet zu kubischen Splines

~cg/2010/skript/Applets/Splines/App.html