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Neuronale Netze (SS 2002)
2. Übungsblatt

Abgabe: Donnerstag (!!!), $25.4.02$, $10^{\underline{15}}$ Uhr, Briefkasten ‚Neuronale Netze' im 4. Stock des AVZ

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1. (5 points) Assume the training set $\{(1,-2,1;0),(-3,1,1;1),(1,1,1;1)\}$ in $\mathbb{R}^3 \times \{0,1\}$ is given and a perceptron with initial weights $(1,2,3)$ (no bias) is trained with the perceptron learning algorithm. Explain all possible steps in one iteration of the perceptron learning algorithm. Does the perceptron algorithm converge for this training set? Which of the following weight vectors can occur as solution (why?): $(-1,0,0)$, $(0,1,0)$, $(0,0.5,0)$.

   And a nasty question: Which of the above vectors can occur as a solution if the algorithm starts in $(0,0,0)$?

2. (5 points) We have neglected the bias in the perceptron learning rule. However, it can be included: Show that the bias can be simulated with an additional weight and extended inputs, i.e. if $\vec{x}$ is input to a neuron with weight $\vec{w}$ and bias θ, then the same computation can be performed with input $(\vec{x},1)$ and neuron with extended weight $(\vec{w},w_{n+1})$ with appropriate w_{n+1}, but no bias.

   What are the consequences for the perceptron learning algorithm, i.e. how would an explicit learning rule which trains weights and bias look like?

3. (5 points) Show the following: Given a finite pattern set which is linearly separable with a perceptron. Then we can find a weight vector $\vec{w}$ and a bias θ such that for each point in the training set the following holds:

$$\begin{array}{ll} \vec{w}^t\vec{x} - \theta \geq 1 & \text{if } \vec{x} \text{ is a positive training example} \\ \vec{w}^t\vec{x} - \theta \leq -1 & \text{if } \vec{x} \text{ is a negative training example} \end{array}$$

   Another nasty question: we have used a vector $\vec{w}$ with the above property in the proof of the perceptron algorithm. Can you think of any benefit if the length of $\vec{w}$ is small?

4. (5 points) Implement the perceptron learning algorithm (you can use an arbitrary programming language) and test it for the logical AND:$\{0,1\}^2 \to \{0,1\}$ and OR:$\{0,1\}^2 \to \{0,1\}$. Please document your program, the training for the two problems, and the respective solution.

   Attention: Don't forget the bias! You can either use the rule for the bias update which you derive in exercise 2. Alternatively you can simulate the bias with an additional weight and extended training patterns as described above, e.g. the training set $(0,0,1;0)$, $(1,0,1;0)$, $(0,1,1;0)$, $(1,1,1;1)$ with additional third component 1 for each pattern is to be trained for AND.