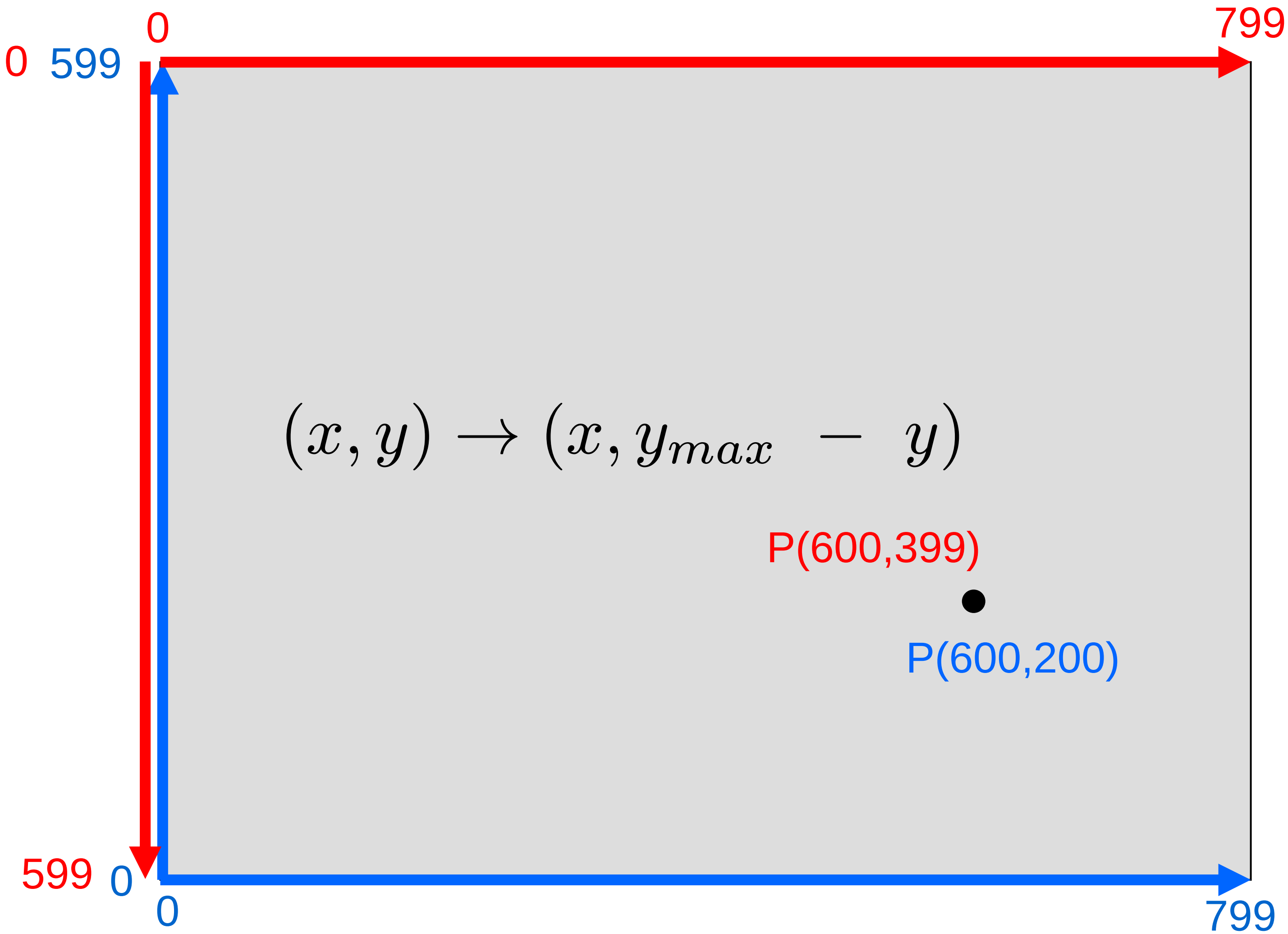
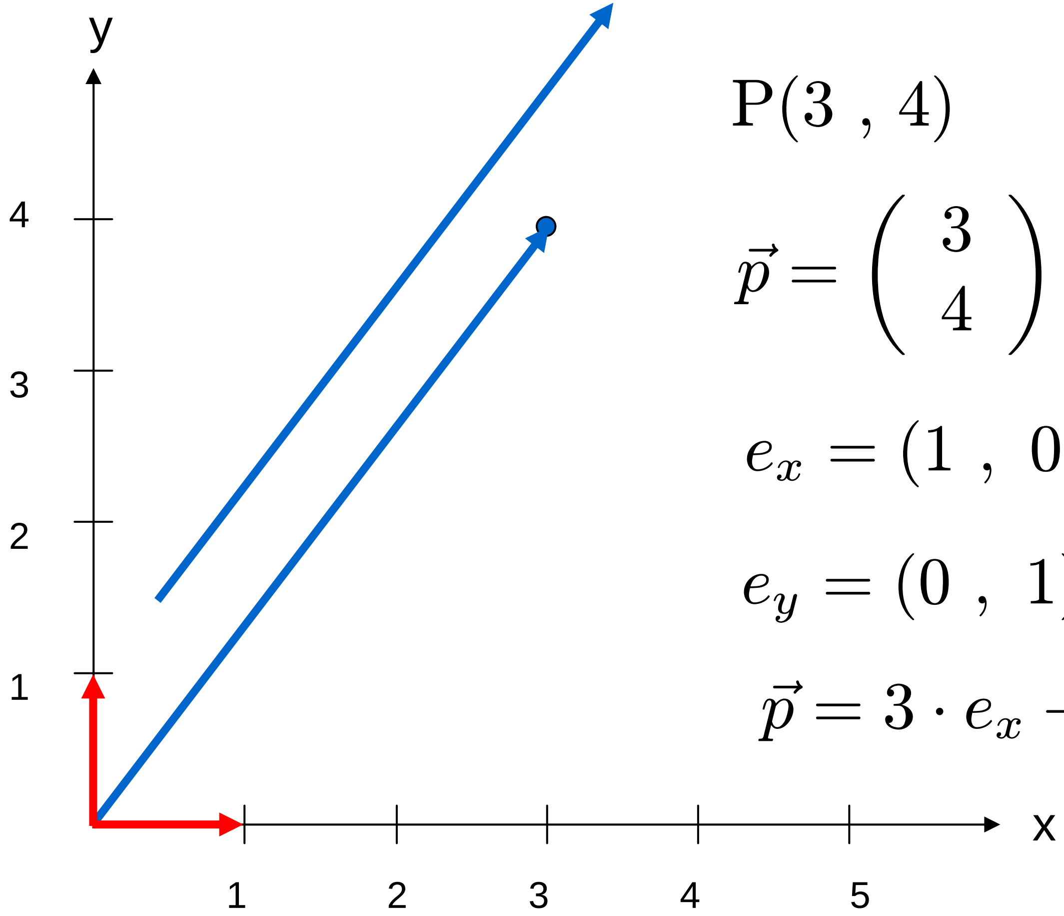
Kapitel 3:
2D-Grundlagen

Koordinatensysteme



$$(x, y) \rightarrow (x, y_{max} - y)$$

Punkt + Vektor



$$P(3\ ,\ 4)$$

$$\vec{p} = \begin{pmatrix} 3 \\ 4 \end{pmatrix} = (3\ ,\ 4)^T$$

$$e_x = (1\ ,\ 0)^T$$

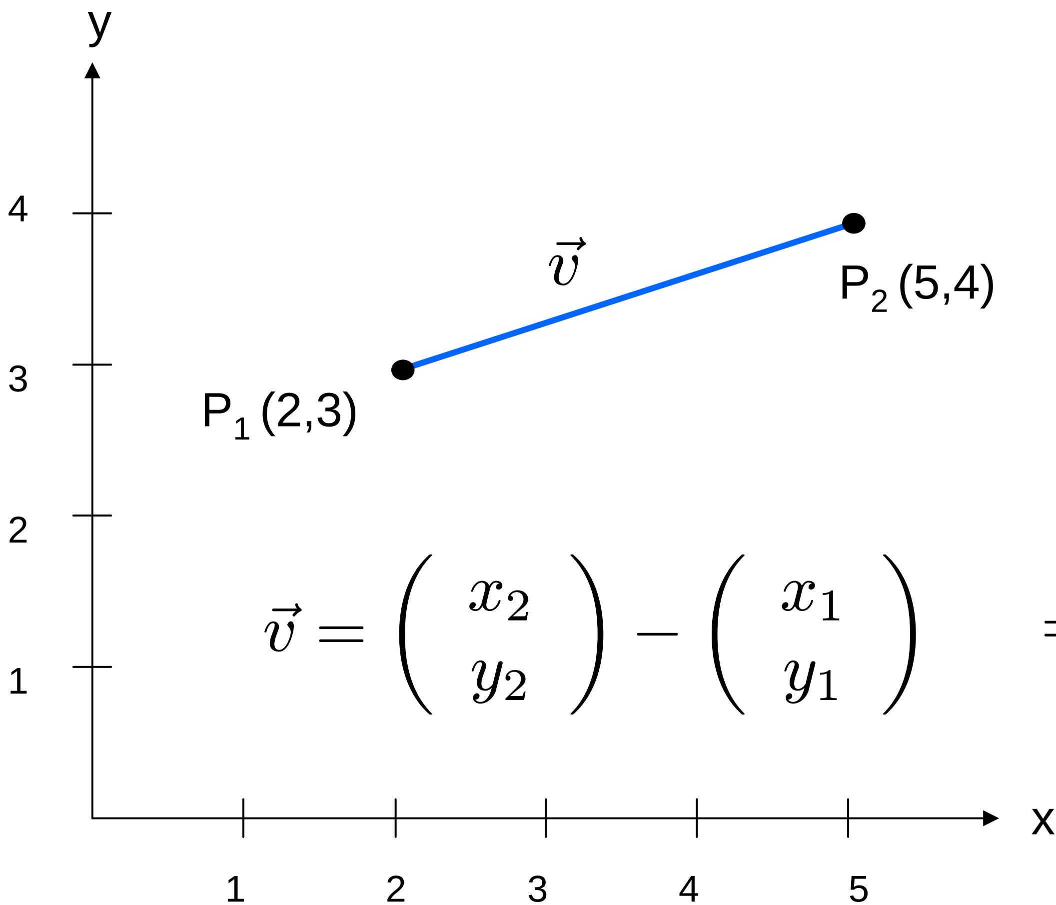
$$e_y = (0\ ,\ 1)^T$$

$$\vec{p} = 3 \cdot e_x + 4 \cdot e_y$$

setPixel(int x, int y)

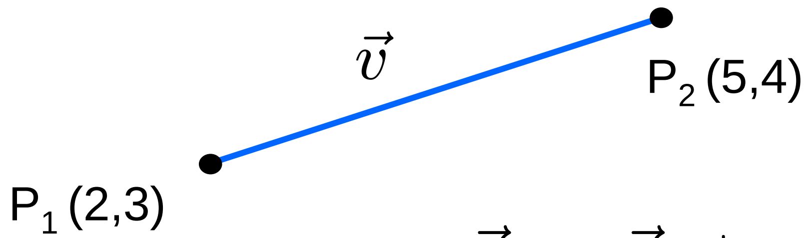
```
setPixel(3,4);

setPixel((int)(x+0.5),(int)(y+0.5));
```

Linie



$$\vec{v} = \begin{pmatrix} x_2 \\ y_2 \end{pmatrix} - \begin{pmatrix} x_1 \\ y_1 \end{pmatrix} = \begin{pmatrix} x_2 - x_1 \\ y_2 - y_1 \end{pmatrix}$$

Parametrisierte Gradengleichung



$$g : \vec{u} = \vec{p_1} + r \cdot \vec{v};\ \ r \in \mathbb{R}$$

$$l : \vec{u} = \vec{p_1} + r \cdot \vec{v};\ \ r \in [0;1]$$

$$d = \|\overline{P_1 P_2}\| \quad = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

VectorLine

```
int x1,y1,x2,y2,x,y,dx,dy;
double r, step;

dy = y2-y1;
dx = x2-x1;

step = 1.0/Math.sqrt(dx*dx+dy*dy);
for (r=0.0; r <= 1; r=r+step) {
  x = (int)(x1+r*dx+0.5);
  y = (int)(y1+r*dy+0.5);
  setPixel(x,y);
}
```

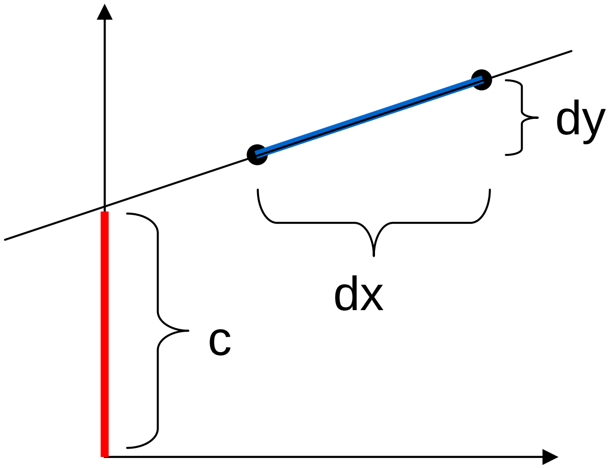
Gradengleichung als Funktion

$$y = f(x) = s \cdot x + c$$

$$s = \frac{y_2 - y_1}{x_2 - x_1}$$

$$\frac{y_1 - c}{x_1 - 0} = \frac{y_2 - y_1}{x_2 - x_1}$$

$$c = \frac{y_1 \cdot x_2 - y_2 \cdot x_1}{x_2 - x_1}$$



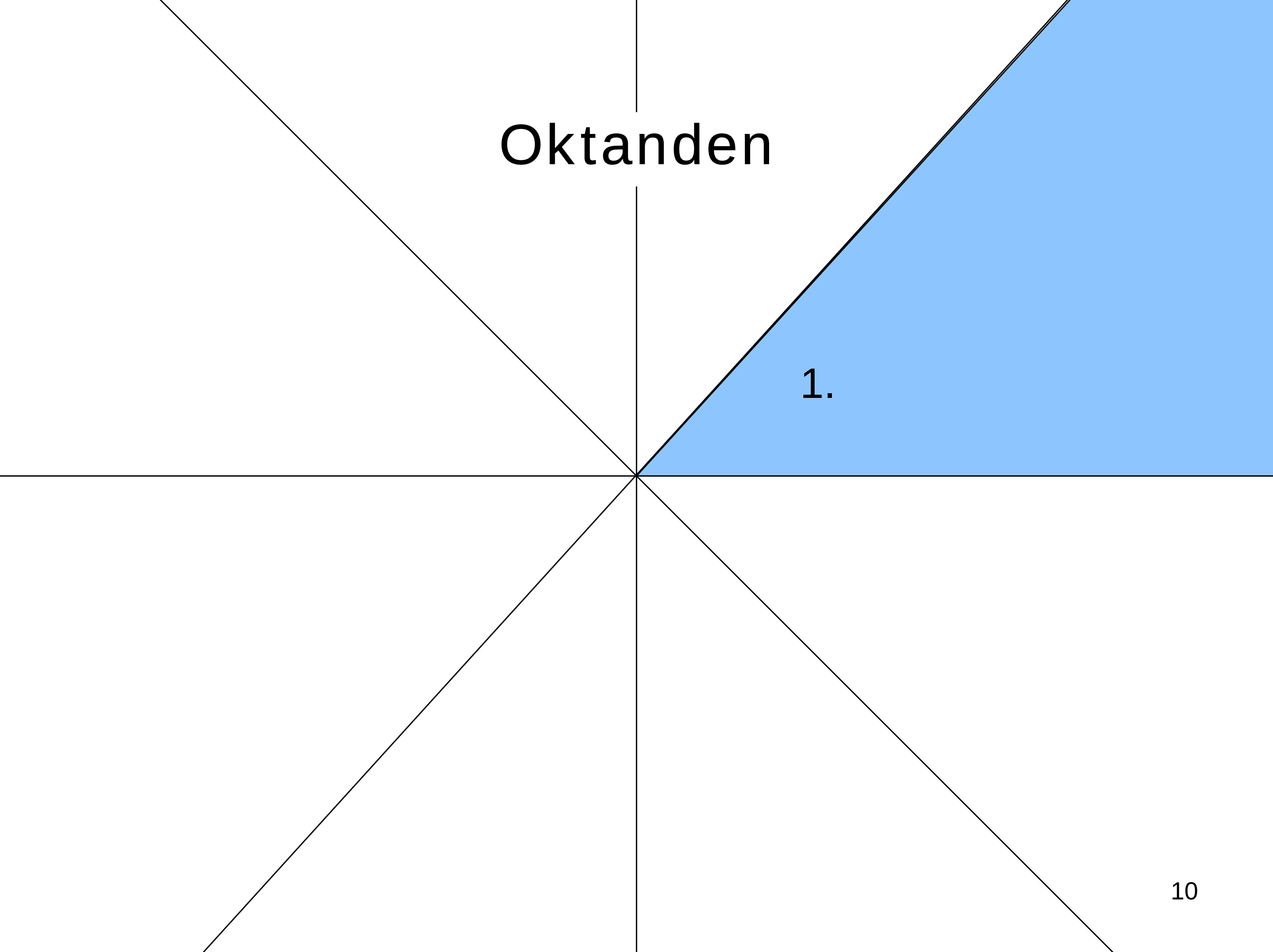
$$y = \frac{y_2 - y_1}{x_2 - x_1} \cdot x + \frac{x_2 \cdot y_1 - x_1 \cdot y_2}{x_2 - x_1}$$

StraightLine

von links nach rechts

```
s = (double)(y2-y1)/(double)(x2-x1);
c = (double)(x2*y1-x1*y2)/(double)(x2-x1);

for (x=x1; x <= x2; x++) {
  y = (int)(s*x+c+0.5);
  setPixel(x,y);
}
```

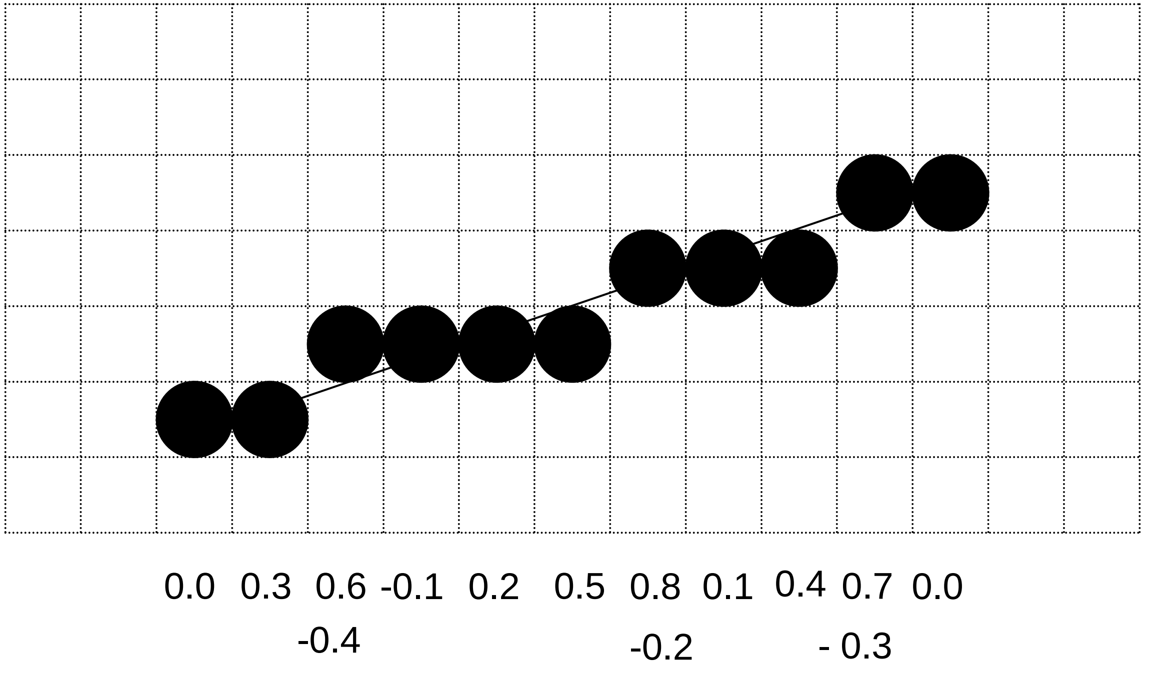
Oktanden



Bresenham

Steigung $s = \Delta y \,/\, \Delta x = 3/10 = 0.3$

Fehler $error = y_{ideal} - y_{real}$



BresenhamLine, die 1.

```
s = (double)(y2-y1)/(double)(x2-x1);
error = 0.0;
x = x1;
y = y1;
while (x <= x2){
  setPixel(x,y);
  x++;
  error = error + s;
  if (error > 0.5) {
    y++;
    error = error – 1.0;
  }
}
```

Integer-Arithmetik

Mache Steigung + Fehler ganzzahlig:

$$s_{neu} = s_{alt} \cdot 2dx = \frac{dy}{dx} \cdot 2dx = 2dy$$

BresenhamLine, die 2.

```
delta = 2*(y2-y1);
error = 0.0;
x = x1;
y = y1;
while (x <= x2){
  setPixel(x,y);
  x++;
  error = error + delta;
  if (error > dx ){
    y++;
    error = error – 2*dx;
  }
}
```

Vergleich mit 0

- vergleiche **`error`** mit 0,

  d.h. verschiebe **`error`** um (x2-x1) nach unten
- verwende **`schritt`** für -2*dx

BresenhamLine, die 3.

```
delta = 2*(y2-y1);
error = -(x2-x1);
schritt = -2*dx
x = x1;
y = y1;
while (x <= x2){
  setPixel(x,y);
  x++;
  error = error + delta;
  if (error > 0 {
    y++;
    error = error + schritt;
  }
}
```

BresenhamLine

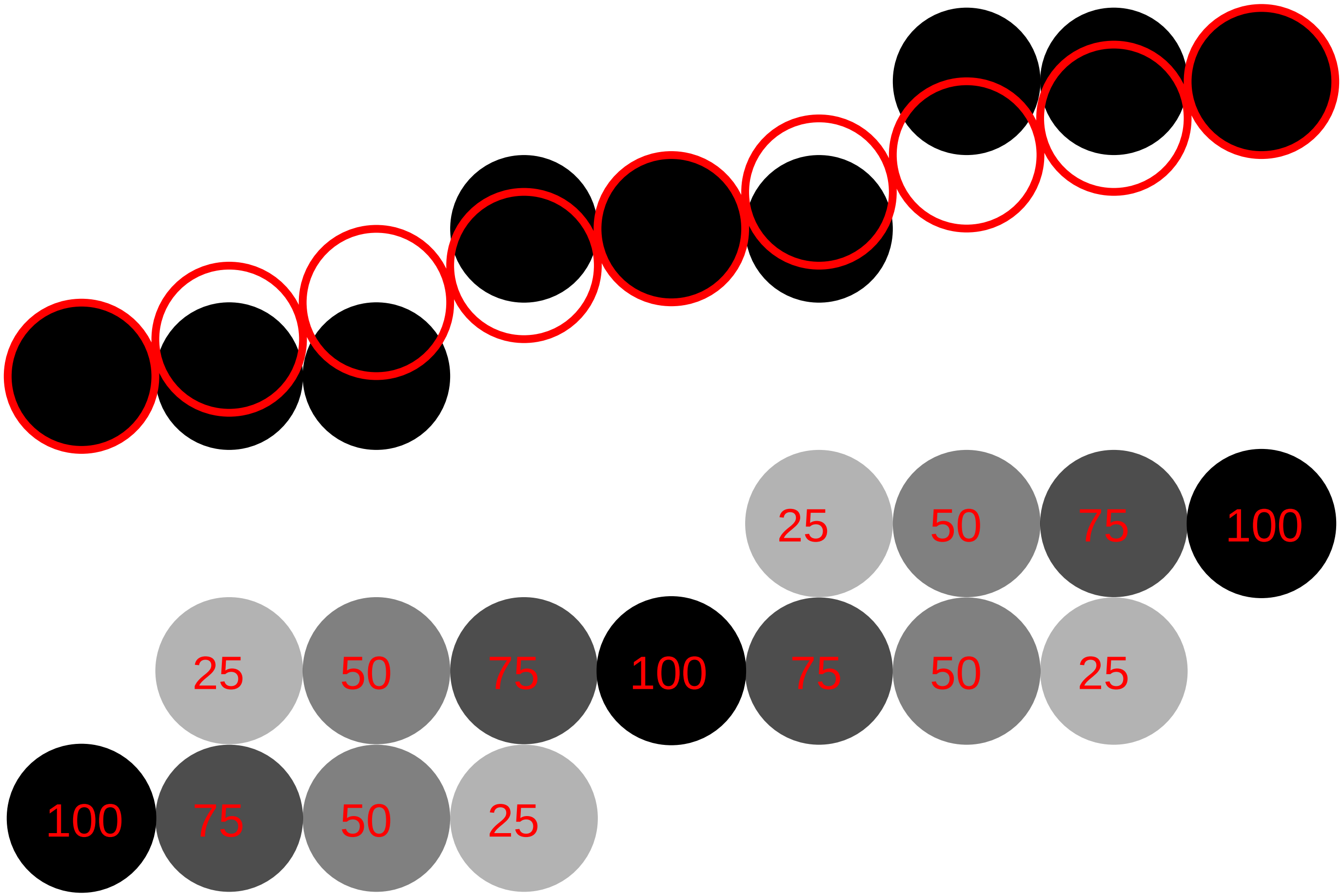
alle 8 Oktanden durch Fallunterscheidung abhandeln:

~cg/2006/skript/Sources/drawBresenhamLine.jav.html

Java-Applet:

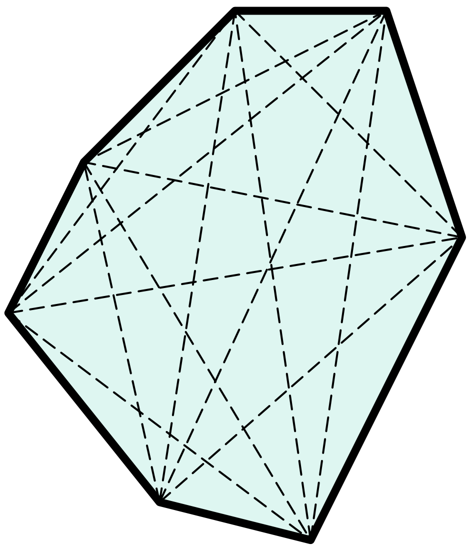
~cg/2006/skript/Applets/2D-basic/App.html

Antialiasing

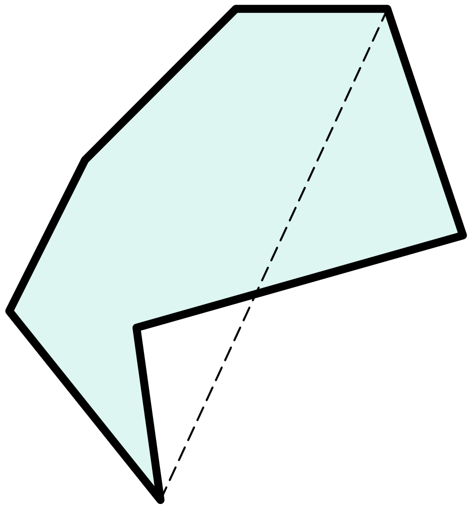


Antialiasing in Adobe Photoshop

Polygon



konvex

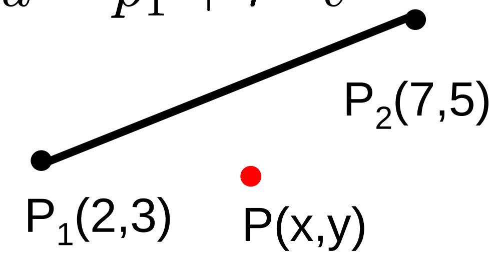


konkav

Punkt versus Gerade

$$\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 2 \\ 3 \end{pmatrix} + r \cdot \begin{pmatrix} 7-2 \\ 5-3 \end{pmatrix}$$

$$\vec{u} = \vec{p_1} + r \cdot \vec{v}$$



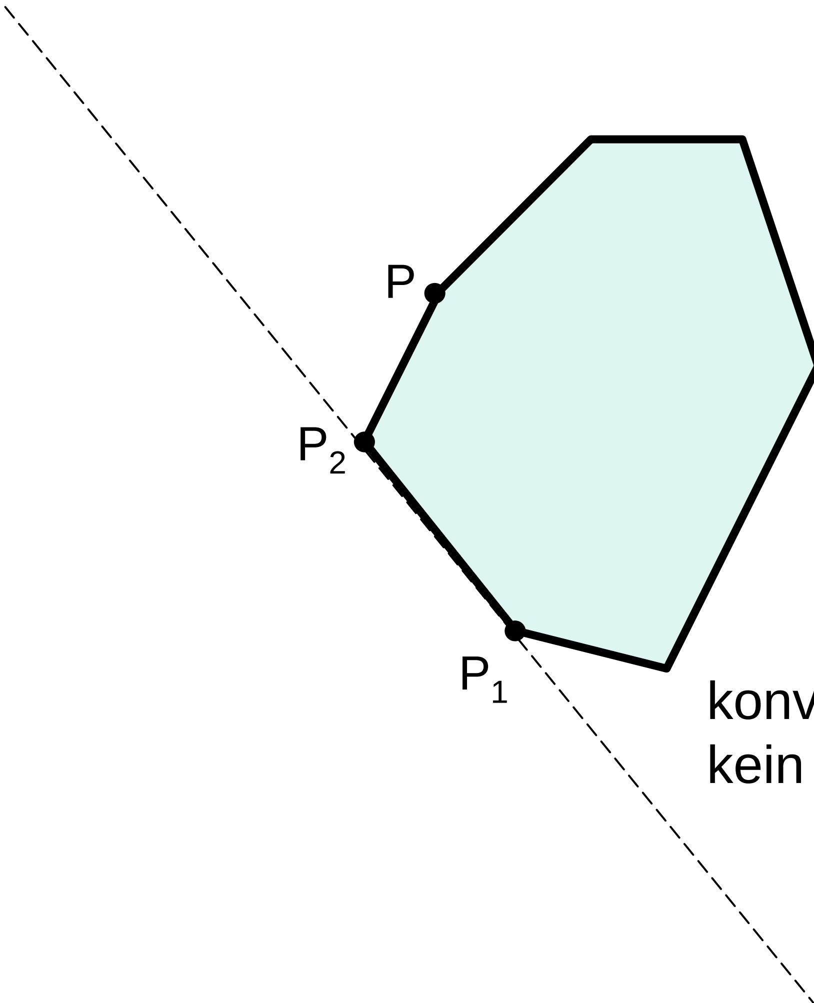
$$\begin{aligned}
x &= 2 + 5r \\
y &= 3 + 2r \\
2x &= 4 + 10r \\
-5y &= -15 - 10r \\
2x - 5y &= -11 \\
2x - 5y + 11 &= 0
\end{aligned}$$

$F(x,y)$ $= 0$ falls P auf der Geraden

< 0 falls P links von der Geraden

> 0 falls P rechts von der Geraden

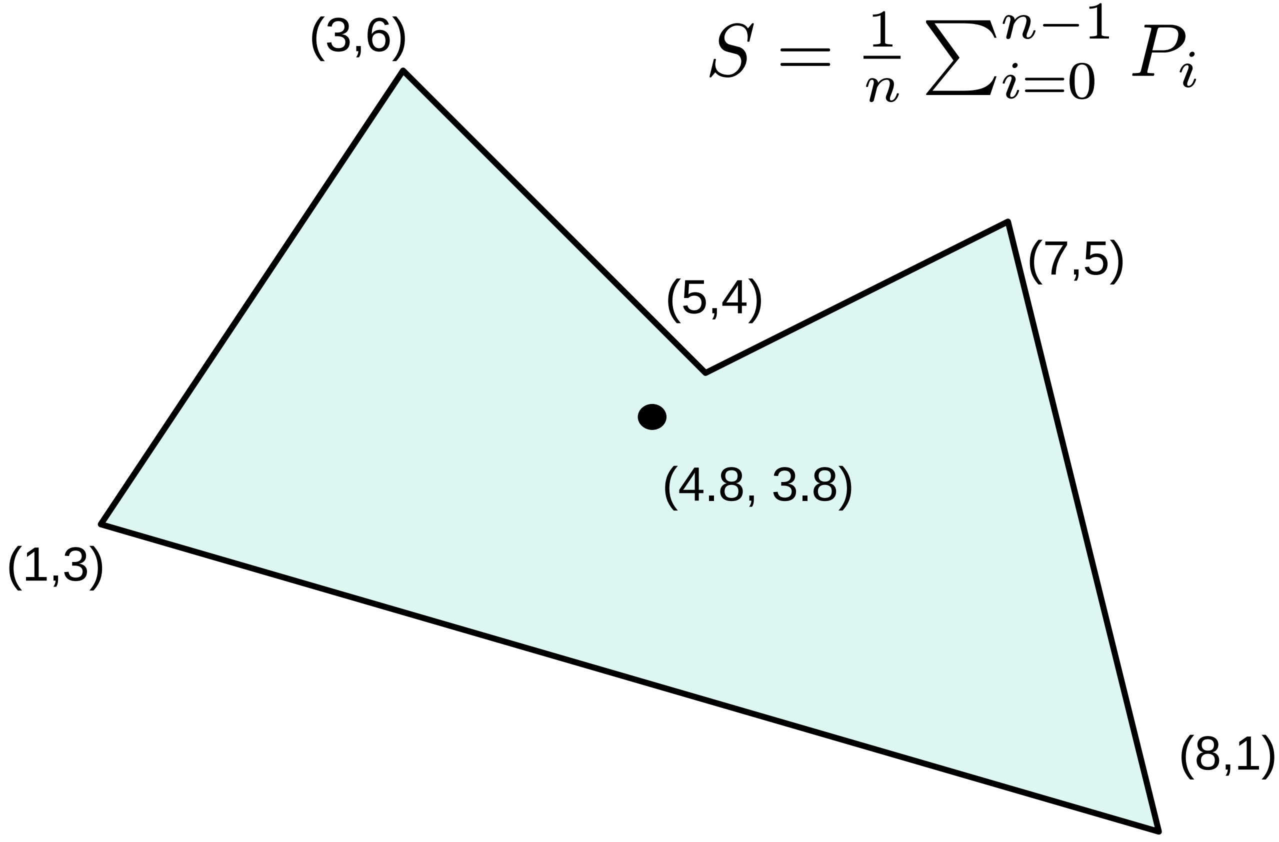
Konvexitätstest nach Paul Bourke



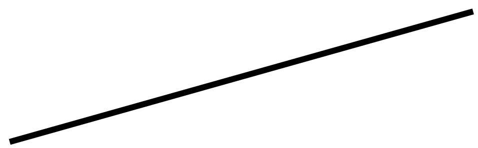
konvex, falls für alle P(x,y)
kein Vorzeichenwechsel bei F(x,y)

Schwerpunkt

$$S = \frac{1}{n} \sum_{i=0}^{n-1} P_i$$



Algorithmen zum Zeichnen



Parametrisiert:

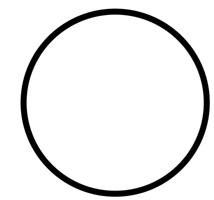
$x := f_1(t); \; y := f_2(t)$

Gradengleichung:

$y := f(x)$

Bresenham:

```
x++; if (…) {y++; … }
```



Parametrisiert:

$x = f_1(t); \; y := f_2(t)$

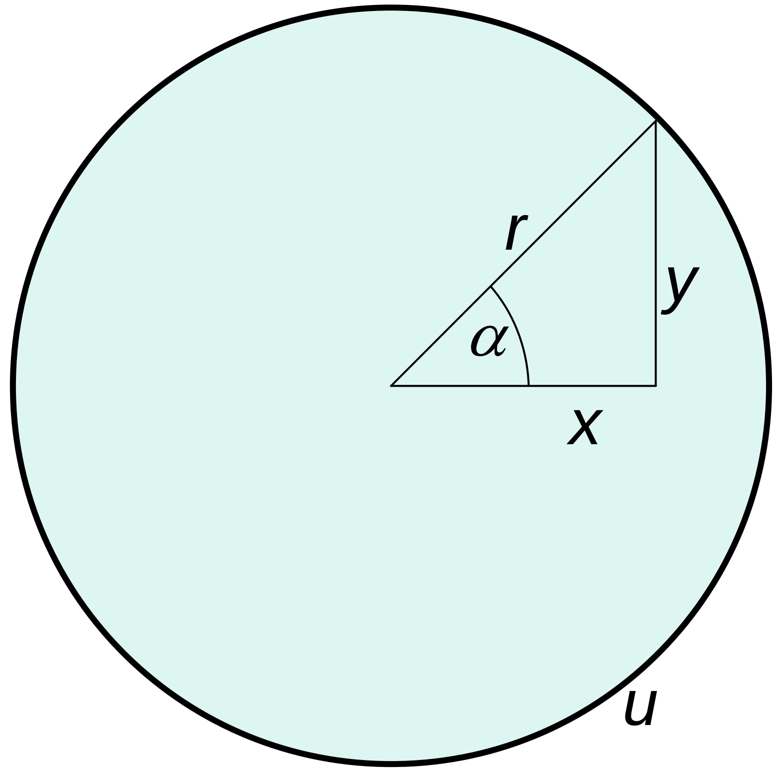
Kreisgleichung:

$y := f(x)$

Bresenham:

```
x++; if (…) {y--; … }
```

Kreis um (0,0), parametrisiert



$x = r \bullet \cos(\alpha)$

$y = r \bullet \sin(\alpha)$

$u = 2 \bullet \pi \bullet r$

$step = 2 \bullet \pi \,/\, 2 \bullet \pi \bullet r = 1/r$

TriCalcCircle

```
double step = 1.0/(double r);
double winkel;

for (winkel = 0.0;
     winkel < 2*Math.PI;
     winkel = winkel+step){

  setPixel((int) r*Math.sin(winkel)+0.5,
           (int) r*Math.cos(winkel)+0.5);
}
```

TriTableCircle

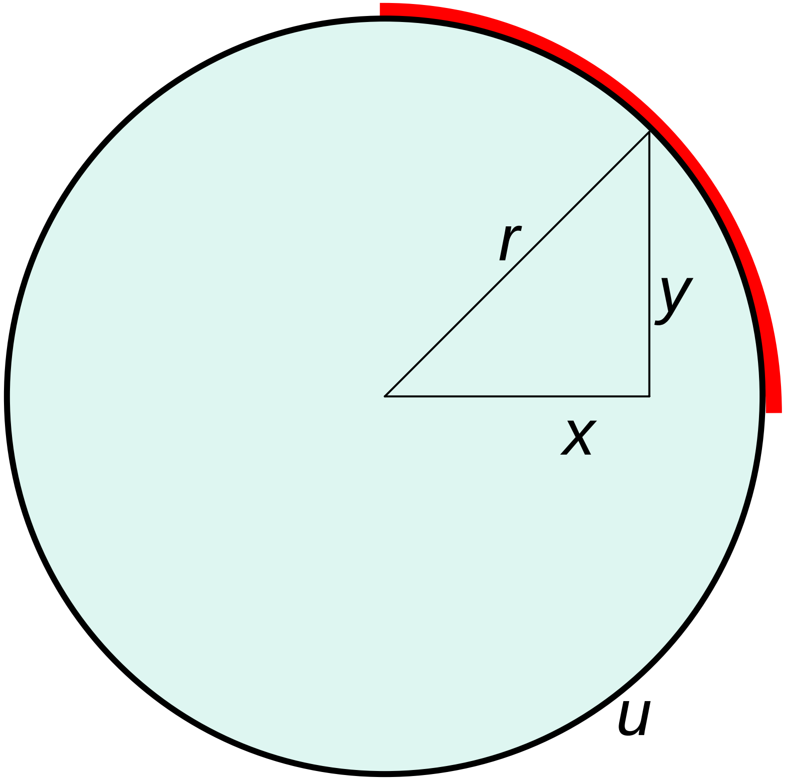
```
// Tabellen sin + cos seien berechnet
// für ganzzahlige Winkel von 0..360

int winkel;

for (winkel = 0;
     winkel < 360;
     winkel++){

  setPixel((int) r*sin[winkel] + 0.5,
           (int) r*cos[winkel] + 0.5);
}
```

Problem: konstante Zahl von Kreispunkten !

Kreis als Funktion im 1. Quadranden

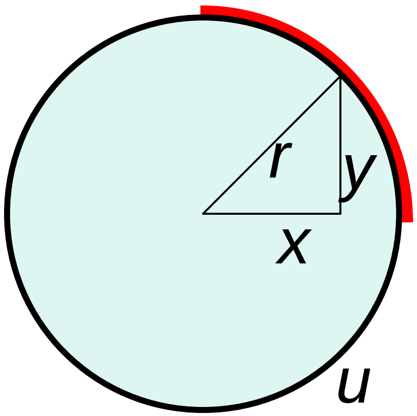
$x^2 + y^2 = r^2$

$$y = \sqrt{r^2 - x^2}$$



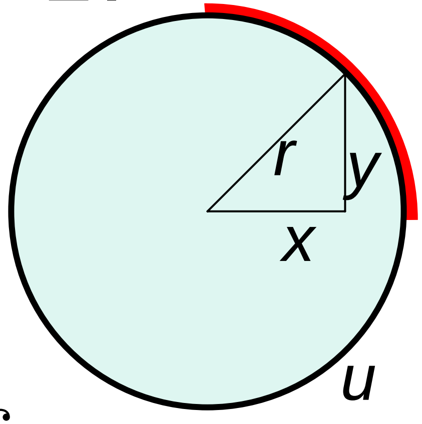
PythagorasCircle, die 1.

$$y = \sqrt{r^2 - x^2}$$




```
for (x=0; x <=r; x++){
  y = (int) Math.sqrt(r*r-x*x);
  setPixel(x,y);
}
```

PythagorasCircle, die 2.

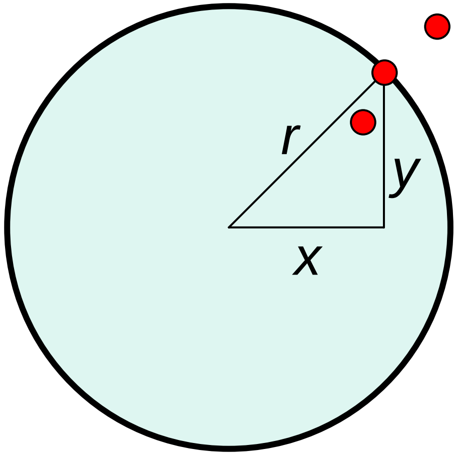


$$y = \sqrt{r^2 - x^2}$$

$$w = r^2 - 1,\ r^2 - 4,\ r^2 - 9,\ r^2 - 16, \ldots$$

```
d = 1;
w = r*r;
for (x=0; x <= r; x++) ){
  y = (int) Math.sqrt(w);
  setPixel(x,y);
  w = w-d
  d = d+2;
}
```

Punkt versus Kreis



$$x^2 + y^2 = r^2$$

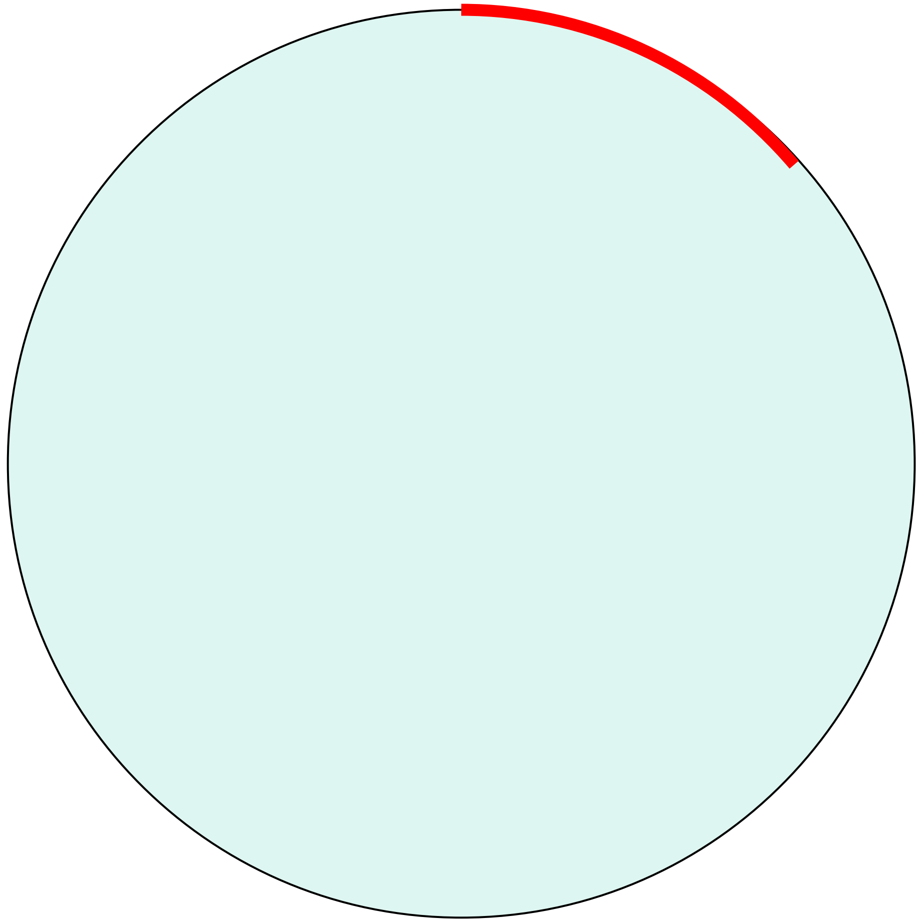
$$F(x,y) = x^2 + y^2 - r^2$$

$F(x,y)$ = 0 für (x,y) auf dem Kreis

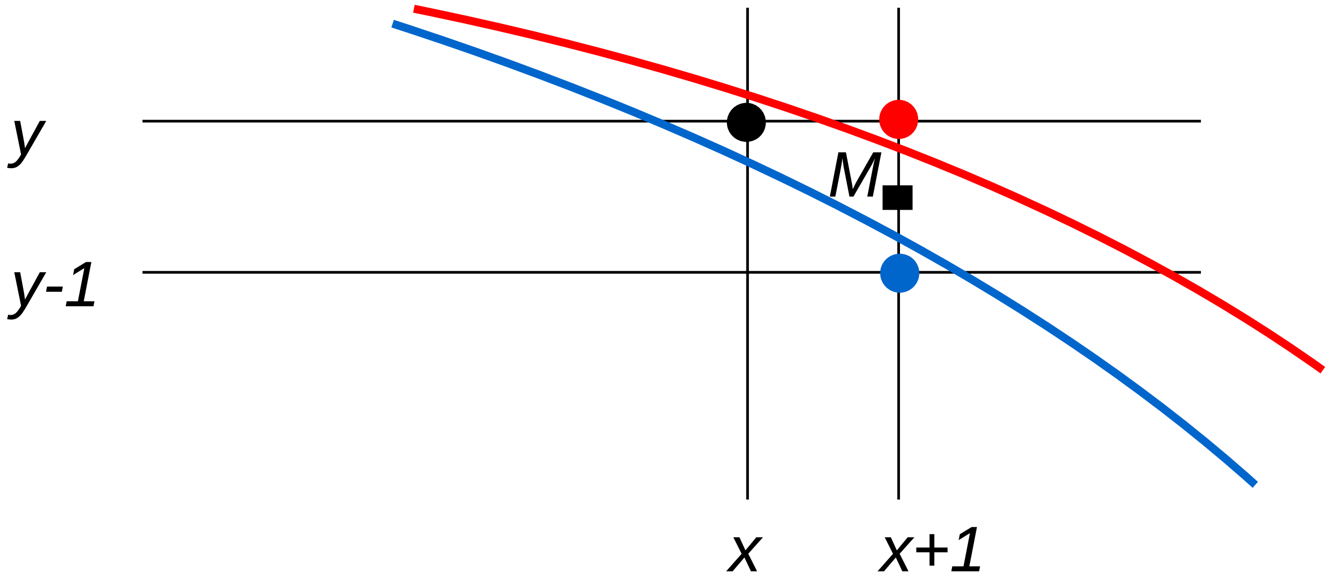
< 0 für (x,y) innerhalb des Kreises

> 0 für (x,y) außerhalb des Kreises

Kreis im 2. Oktanden



Entscheidungsvariable Δ



$\Delta = F(x+1, y-½)$

$\Delta < 0 \Rightarrow M$ liegt innerhalb $\Rightarrow$ wähle $(x+1, y)$

$\Delta \geq 0 \Rightarrow M$ liegt außerhalb $\Rightarrow$ wähle $(x+1, y-1)$

Berechnung von Δ

$\Delta = F(x+1, y-½) = (x+1)^2+(y-½)^2-r^2$

$\Delta < 0 \Rightarrow$

$\Delta' = F(x+2, y-½) = (x+2)^2+(y-½)^2-r^2 =$

$\Delta+2x+3$

$\Delta \geq 0 \Rightarrow$

$\Delta' = F(x+2, y-3/2) = (x+2)^2+(y-3/2)^2-r^2 =$

$\Delta+2x-2y+5$

Startwert $\Delta = F(1, r-½) = 1^2+(r-½)^2 - r^2 =$

$5/4-r$

BresenhamCircle, die 1.

```
x = 0;
y = r;
delta = 5.0/4.0 - r;
while (y >= x) {
  setPixel(x,y);
  if (delta < 0.0) {
    delta = delta + 2*x + 3.0;
    x++;
  else {
    delta = delta + 2*x - 2*y + 5.0;
    x++;
    y--;
  }
}
```

Substitutionen

$d := delta - \frac{1}{4}$

$dx := 2x + 3$

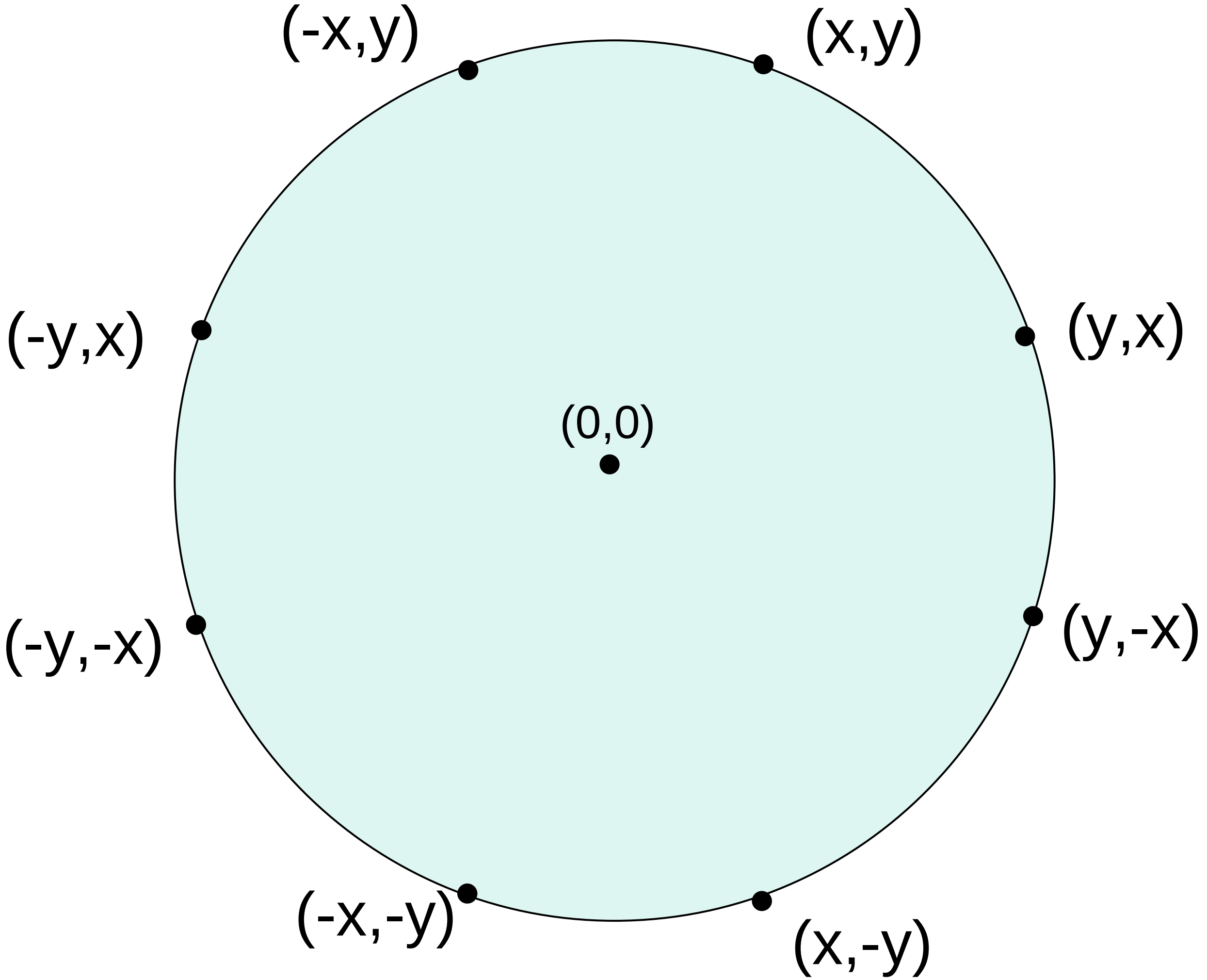
$dxy := 2x - 2y + 5$

BresenhamCircle, die 2.

```
x = 0;
y = r;
~~delta = 5.0/4.0 - r;~~          d = 1 - r;
                                   dx = 3;
                                   dxy = -2*r + 5;
while (y >= x) {
  setPixel(x,y);
  if (~~delta < 0.0~~) {           (d < 0.0)
    ~~delta = delta + 2*x + 3.0~~;   d = d + dx;
    x++;                             dx = dx + 2;
  else {                             dxy = dxy + 2;
    ~~delta = delta+2*x-2*y+5.0~~;   d   = d + dxy;
    x++;                             dx  = dx + 2;
    y--;                             dxy = dxy + 4;
  }
}
```

d:=delta-1/4 dx:=2x+3 dxy:= 2x-2y+5

Oktanden-Symmetrie



BresenhamCircle, die 3.

```
x=0; y=r; d=1-r; x=3; dx=3; dxy=-2*r+5;

while (y>=x){

   setPixel(+x,+y);
   setPixel(+y,+x);
   setPixel(+y,-x);
   setPixel(+x,-y);
   setPixel(-x,-y);
   setPixel(-y,-x);
   setPixel(-y,+x);
   setPixel(-x,+y);

  if (d<0) {d=d+dx;  dx=dx+2; dxy=dxy+2; x++;}
  else     {d=d+dxy; dx=dx+2; dxy=dxy+4; x++; y--;}
}
```

BresenhamCircle

Zahl der erzeugten Punkte = $4 \cdot \sqrt{2} \cdot r$

= 10% unterhalb von $2 \cdot \pi \cdot r$

Kreis mit Radius r um Mittelpunkt (x,y):

~cg/2006/Sources/drawBresenhamCircle.jav

Java-Applet:

~cg/2006/skript/Applets/2D-basic/App.html

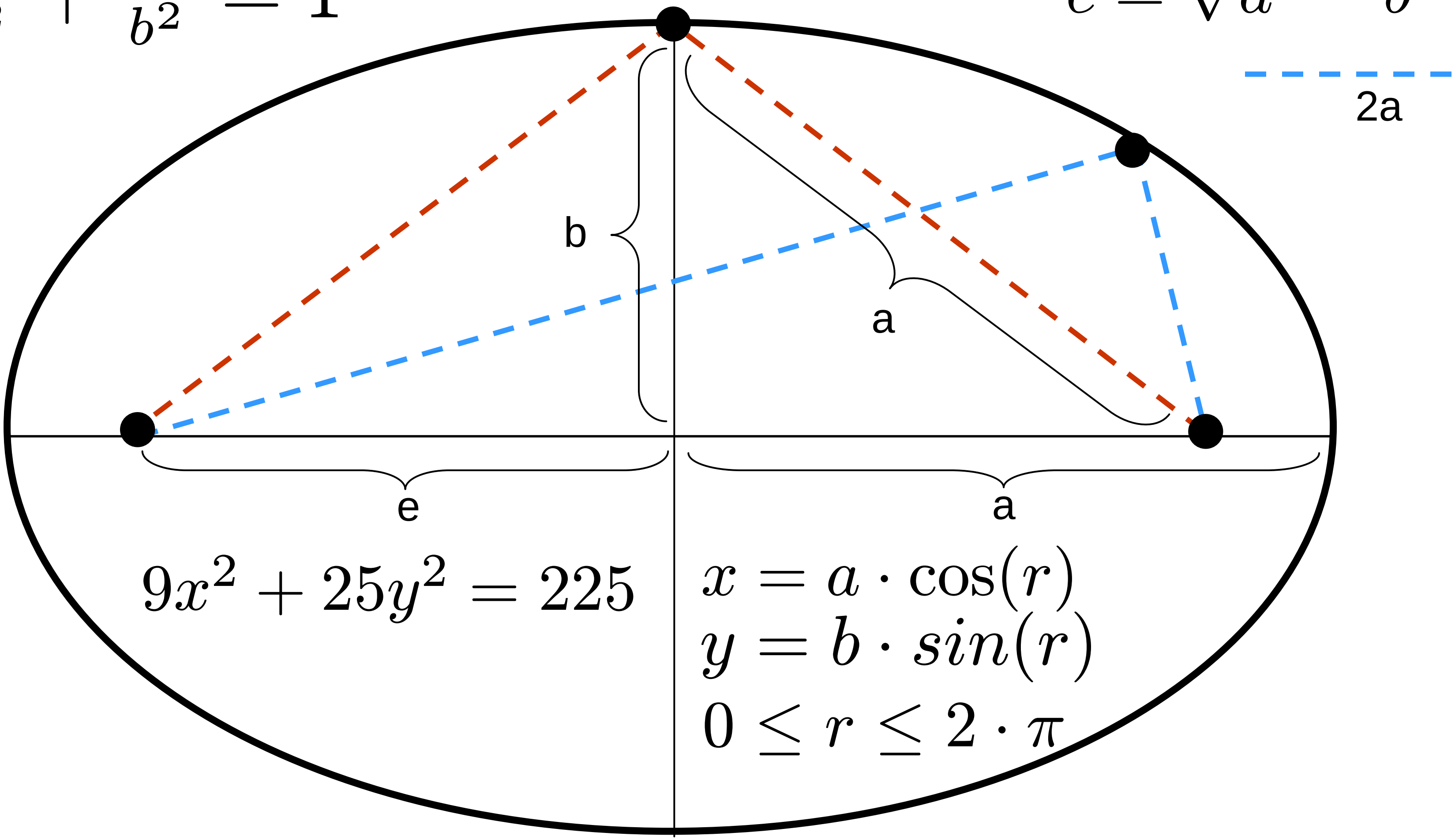
Java-Applet für Performance-Messung

~cg/2006/skript/Applets/circle/App.html

Ellipse um (0,0)

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

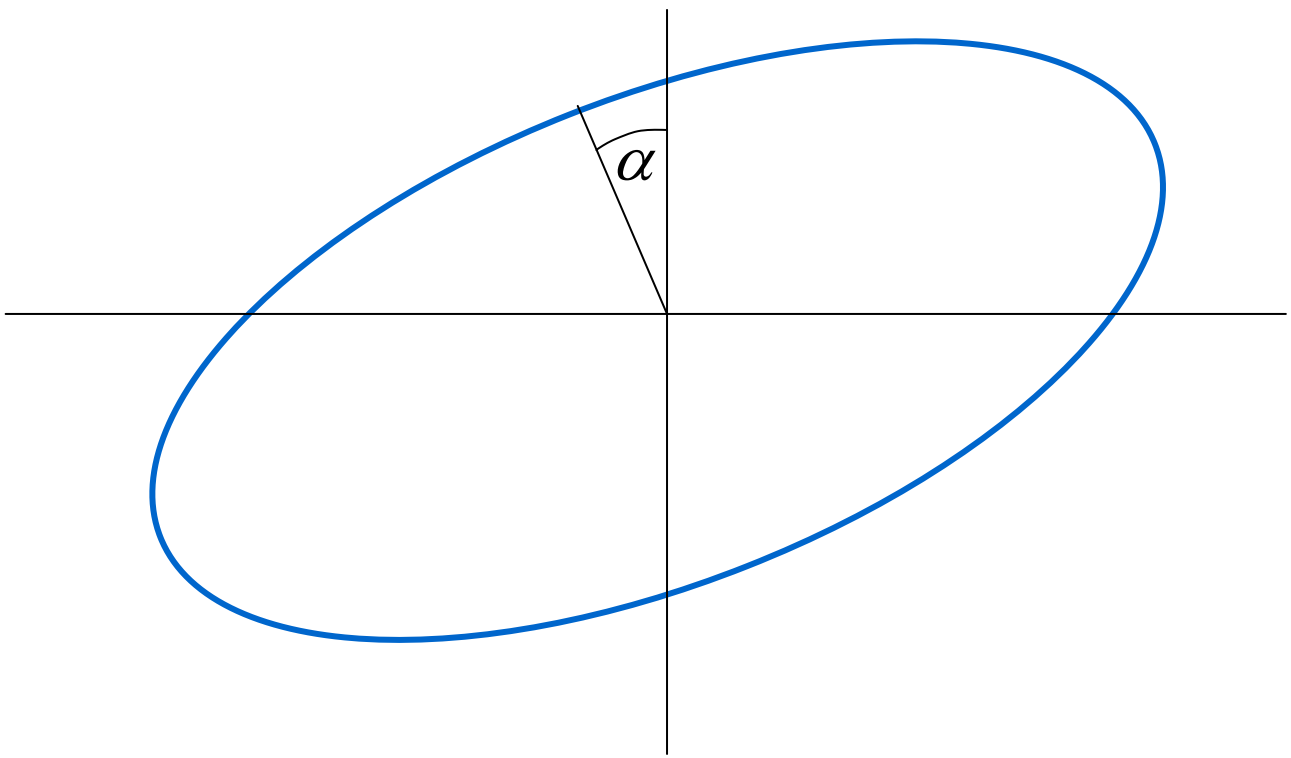
$$e = \sqrt{a^2 - b^2}$$



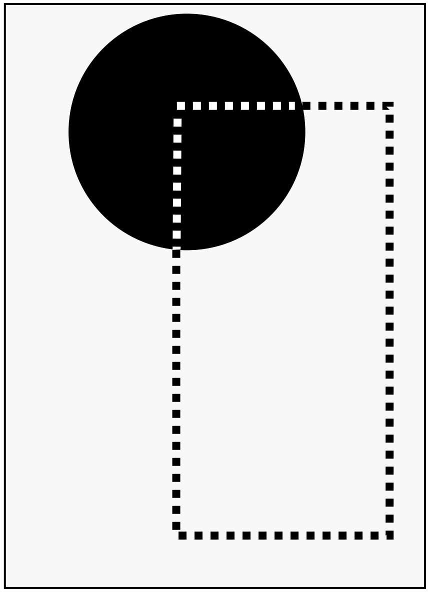
$$9x^2 + 25y^2 = 225$$

$$x = a \cdot \cos(r)$$
$$y = b \cdot sin(r)$$
$$0 \leq r \leq 2 \cdot \pi$$

Ellipse, gedreht



Zeichnen und Löschen mit XOR



Pixel:	01101011
Gummiband:	11111111
XOR ergibt:	10010100
Gummiband:	11111111
XOR ergibt:	01101011

Beispiel für Gummiband:
~cg/2006/skript/Applets/2D-basic/App.html