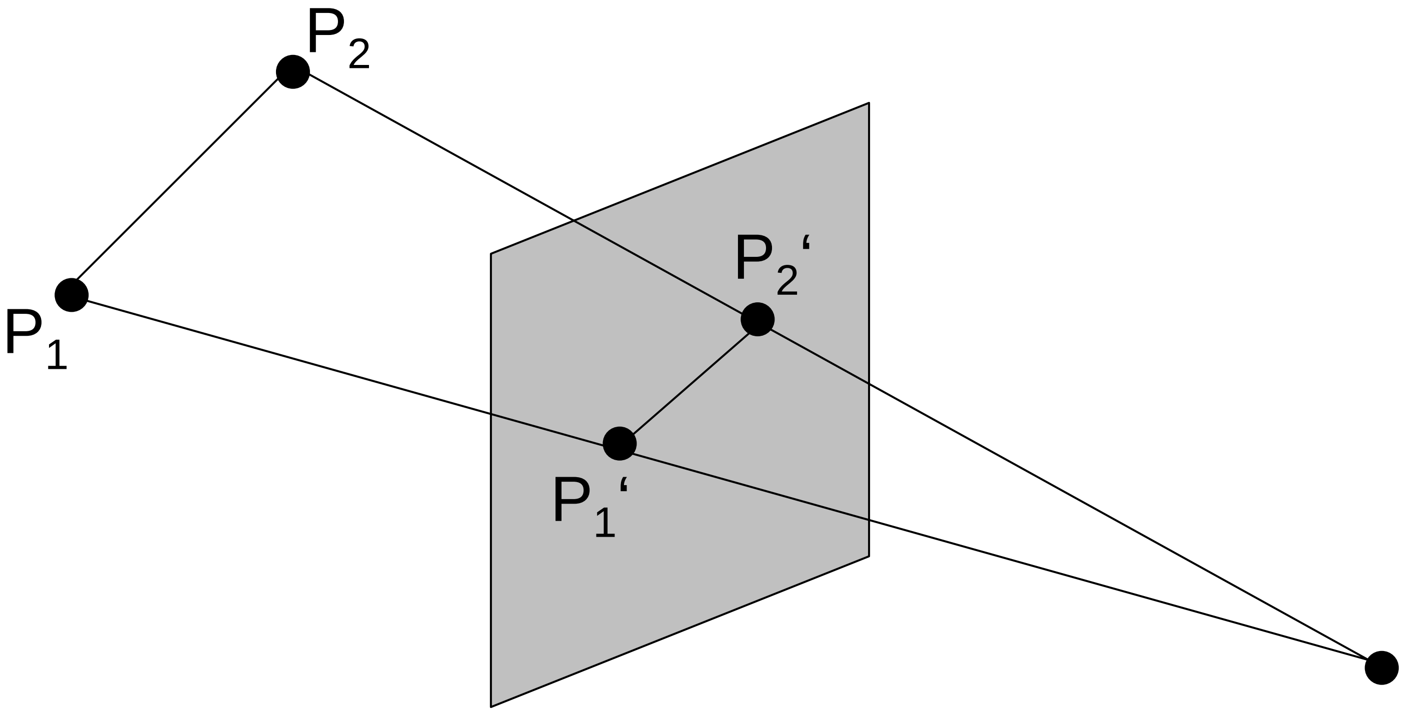
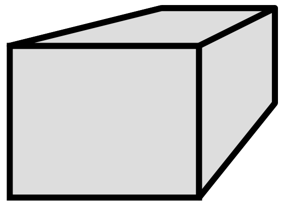
Kapitel 14: Projektion

Projektion

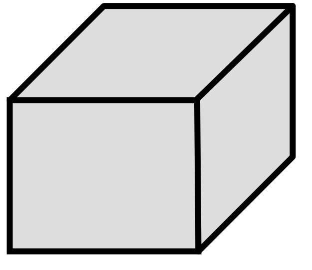


Projektionsarten

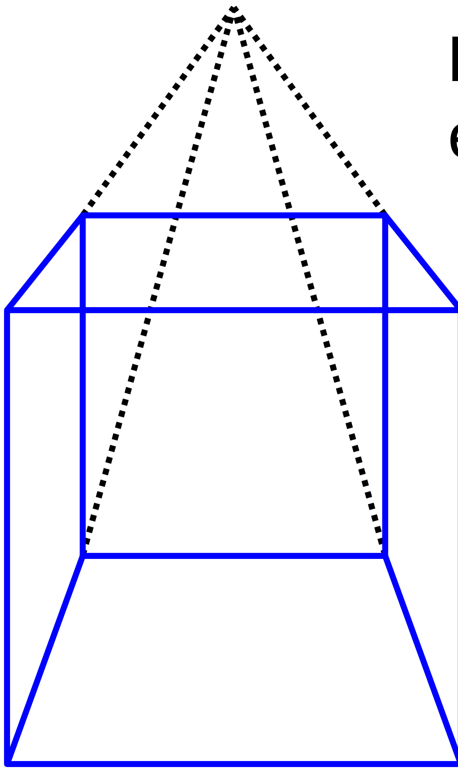
- Zentralprojektion:
  Augenpunkt im endlichen Abstand



- Parallelprojektion:
  Augenpunkt im Unendlichen



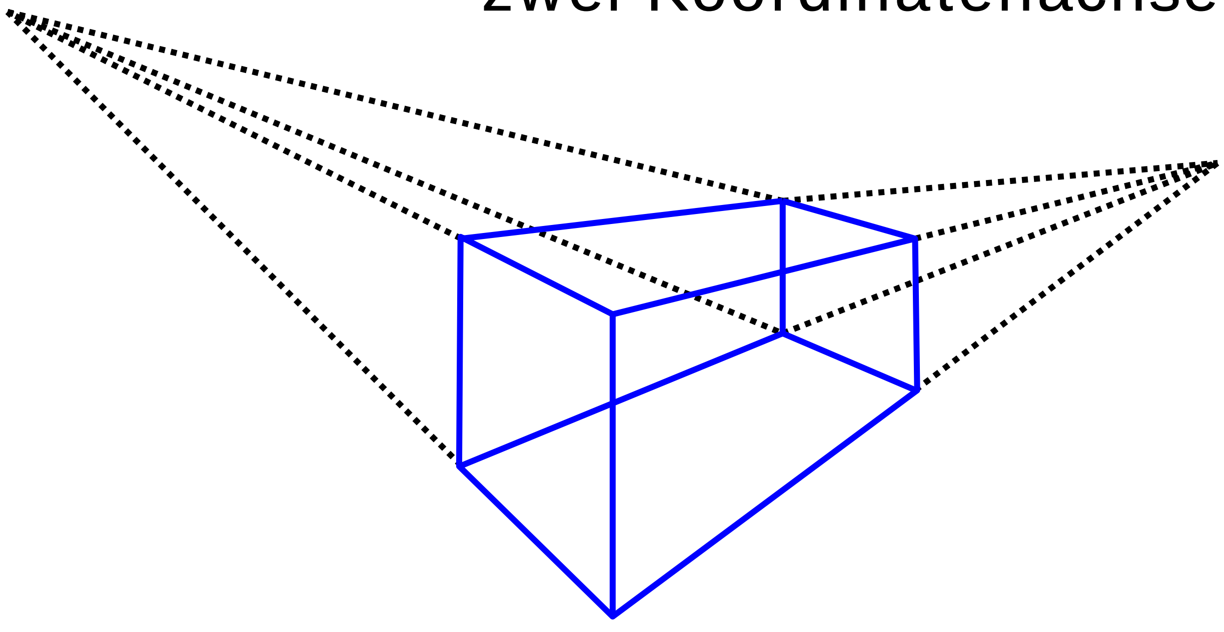
1 Fluchtpunkt



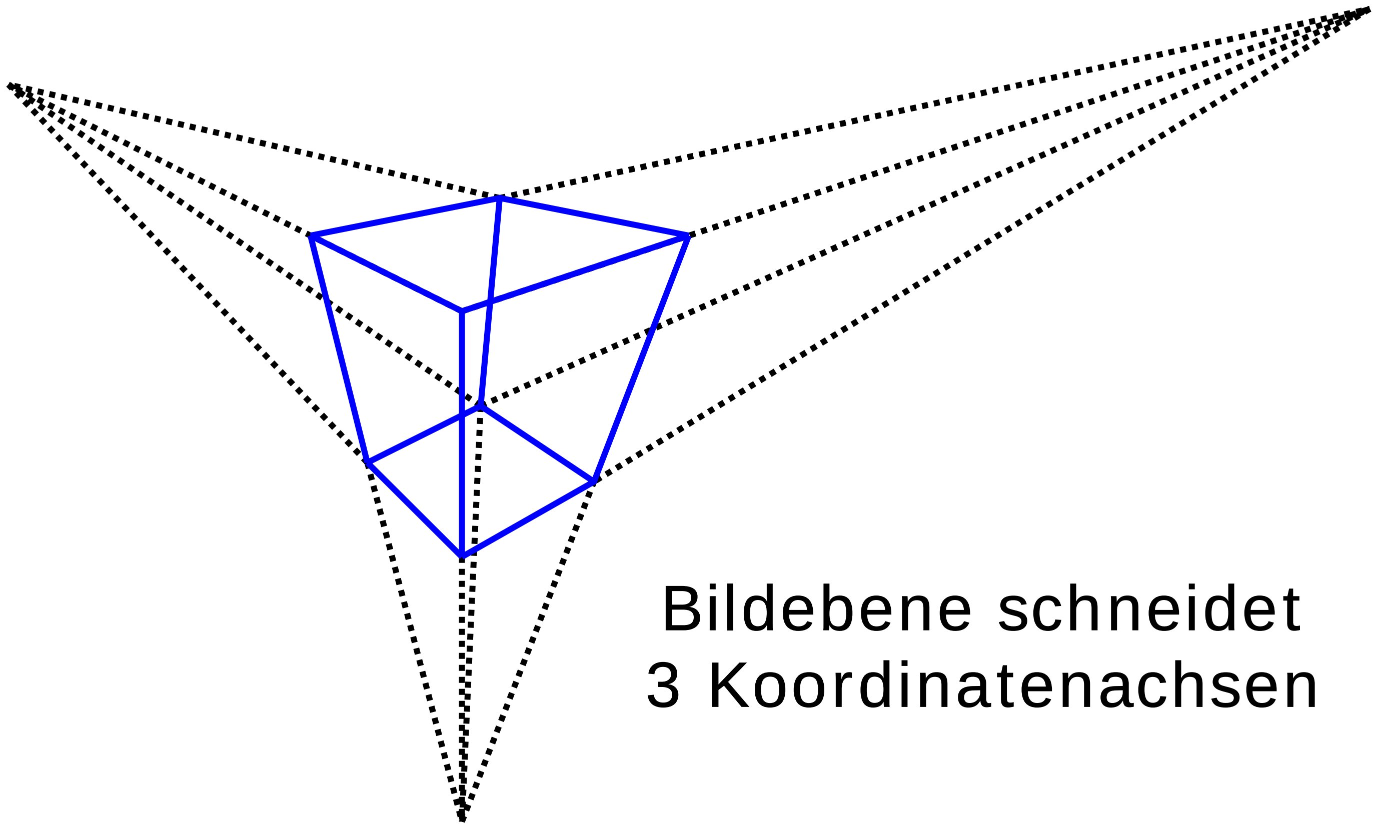
Bildebene schneidet
eine Koordinatenachse

2 Fluchtpunkte

Bildebene schneidet
zwei Koordinatenachsen



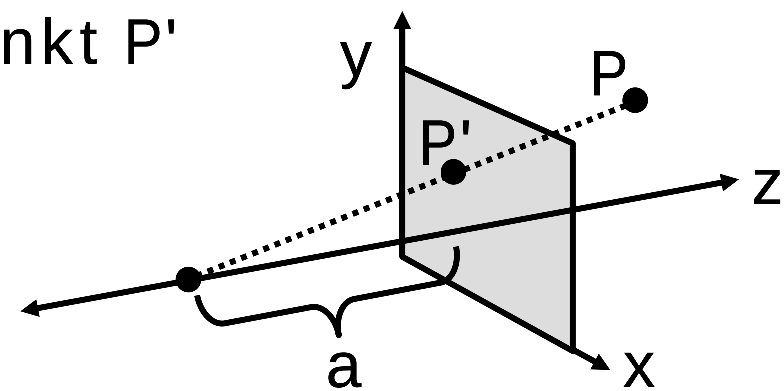
3 Fluchtpunkte



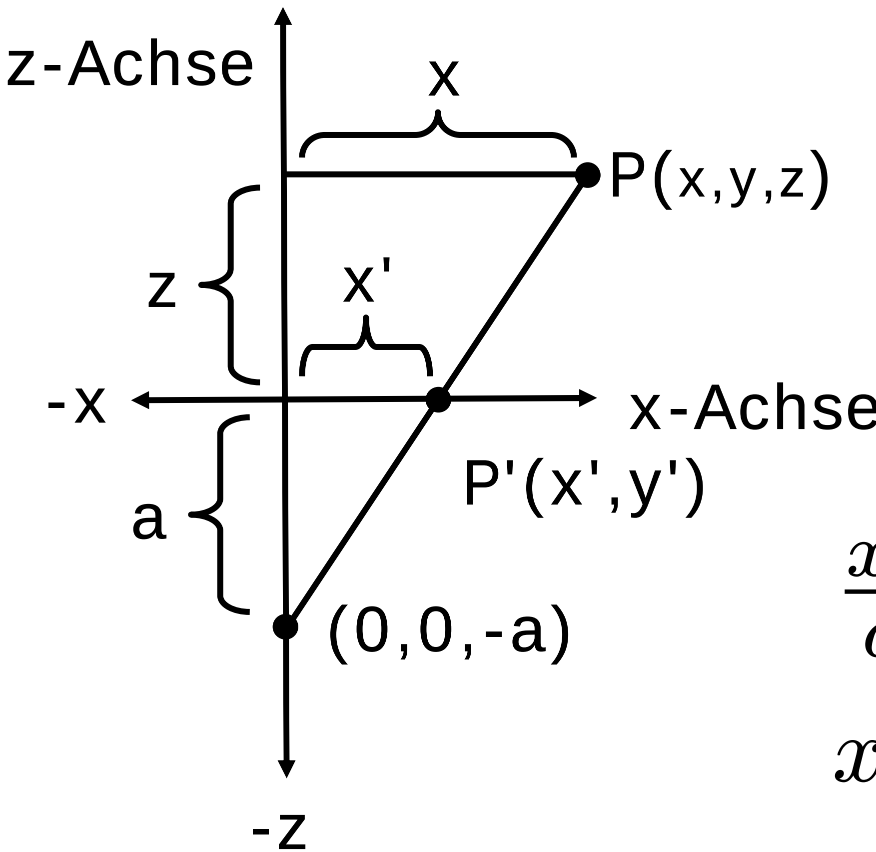
Bildebene schneidet
3 Koordinatenachsen

Aufgabenstellung

- Bildebene sei in xy-Ebene
- Augenpunkt sei auf negativer z-Achse bei -a
- Gegeben Punkt P
- Finde Schnittpunkt P'



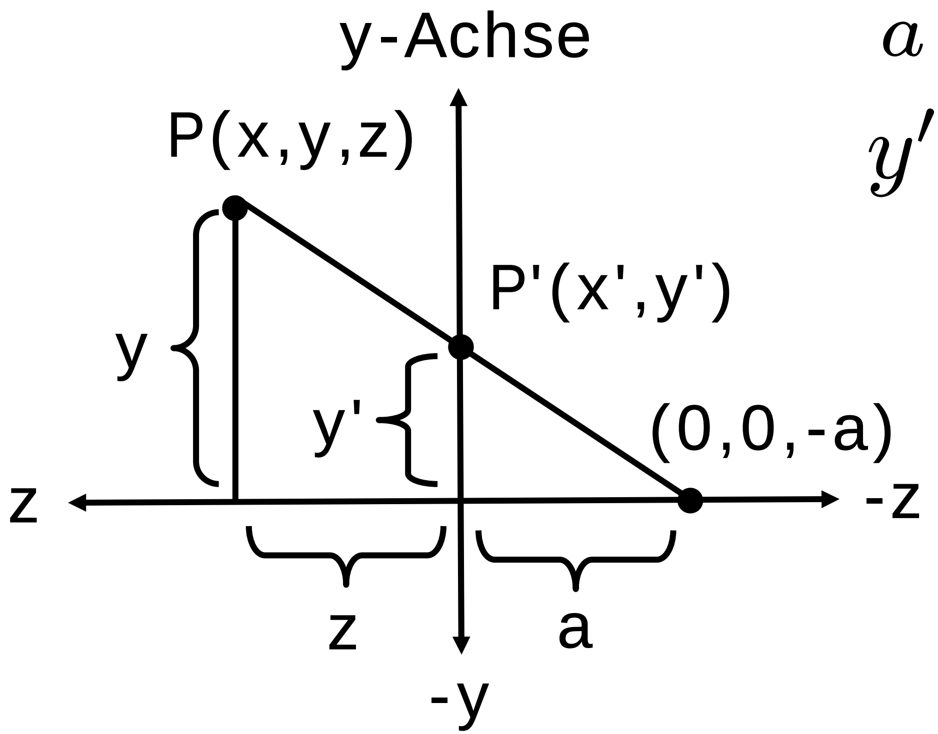
Blick von oben



$$\frac{x'}{a} = \frac{x}{a+z}$$

$$x' = \frac{x}{1+z/a}$$

Blick von der Seite



$$\frac{y'}{a} = \frac{y}{a+z}$$

$$y' = \frac{y}{1+z/a}$$

Ergebnis

$$x' = \frac{x}{1+z/a} \qquad y' = \frac{y}{1+z/a}$$ z merken

$$x' = \frac{x}{w} \qquad y' = \frac{y}{w} \qquad w = 1 + z/a$$

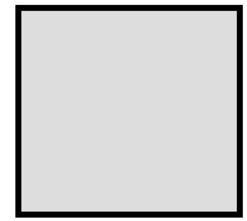
$$P' = \left(\frac{x}{w}, \frac{y}{w}, 0, 1\right) \quad = (x,\ y,\ 0,\ 1 + z/a)$$

$$\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 1/a & 1 \end{pmatrix} \cdot \begin{pmatrix} x \\ y \\ z \\ 1 \end{pmatrix} = \begin{pmatrix} x \\ y \\ 0 \\ 1+z/a \end{pmatrix}$$

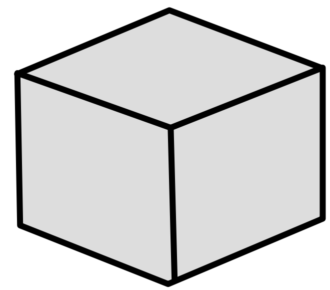
Parallelprojektion

Normalprojektion

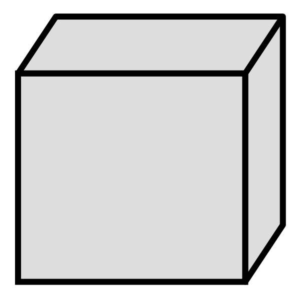
Grund-,Seiten-,Aufriss:



axonometrische Projektion:



schiefe Projektion



Normalprojektion

Bilde $(x,y,z,1)$ auf $(x,y,0,1)$ ab:

$$P_{ortho_{xy}} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

Schiefe Projektion

$x' = x - L \cdot \cos(\alpha)$

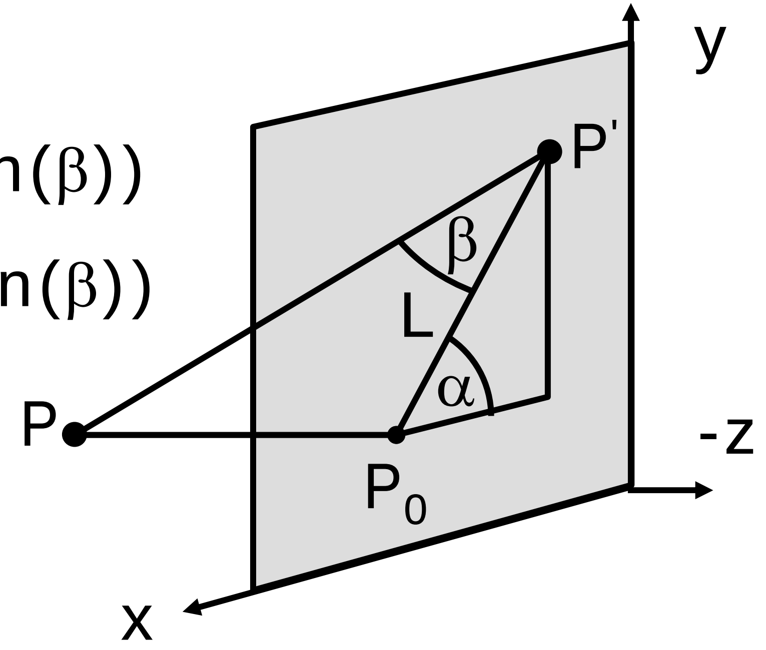
$y' = y + L \cdot \sin(\alpha)$

$z' = 0$

$x' = x - z \cdot (\cos(\alpha) / \tan(\beta))$

$y' = y + z \cdot (\sin(\alpha) / \tan(\beta))$

$\tan(\beta) = z / L$



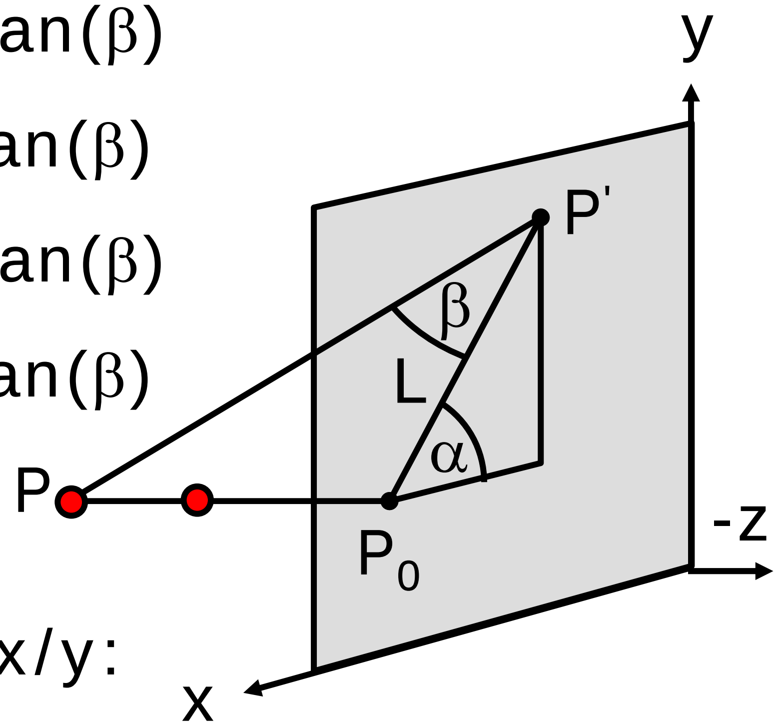
x-Ausdehnung zu z-Ausdehnung

$x'_1 = x_1 - z_1 \bullet \cos(\alpha)/\tan(\beta)$

$y'_1 = y_1 + z_1 \bullet \sin(\alpha)/\tan(\beta)$

$x'_2 = x_2 - z_2 \bullet \cos(\alpha)/\tan(\beta)$

$y'_2 = y_2 + z_2 \bullet \sin(\alpha)/\tan(\beta)$



2 Punkte auf Lot zu x/y:

$|x'_1 - x'_2| = |(z_1 - z_2) \bullet \cos(\alpha)/\tan(\beta)|$

$|y'_1 - y'_2| = |(z_1 - z_2) \bullet \sin(\alpha)/\tan(\beta)|$

Verkürzungsfaktor

$$|P_1' - P_2'| = \sqrt{|x_1' - x_2'|^2 + |y_1' - y_2'|^2}$$

$$|P_1' - P_2'| = \sqrt{\frac{(z_1 - z_2)^2}{\tan^2(\beta)} \cdot (\cos^2(\alpha) + \sin^2(\alpha))}$$

$$\cos^2(\alpha) + \sin^2(\alpha) = 1$$

$$= \frac{z_1 - z_2}{\tan(\beta)}$$

$$d = \frac{1}{\tan(\beta)}$$

$d = 1 \quad \Rightarrow \quad \beta = 45°$

$d = 0.5 \quad \Rightarrow \quad \beta = 63.43°$

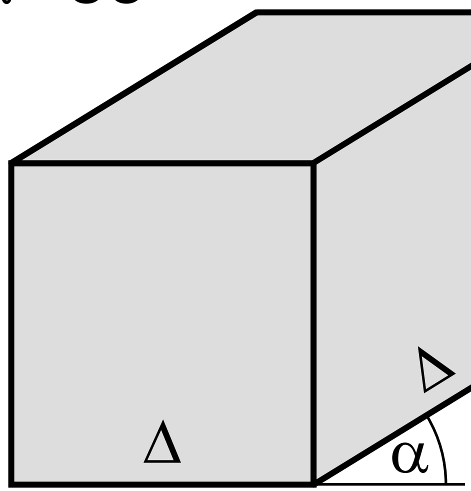
schiefe Transformationsmatrix

$$x' = x - z \cdot (d \cdot \cos(\alpha))$$
$$y' = y + z \cdot (d \cdot \sin(\alpha))$$
$$z' = 0$$
$$w' = 1$$

$$P_{schief_{xy}}(\alpha, d) = \begin{pmatrix} 1 & 0 & -d \cdot \cos(\alpha) & 0 \\ 0 & 1 & d \cdot \sin(\alpha) & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

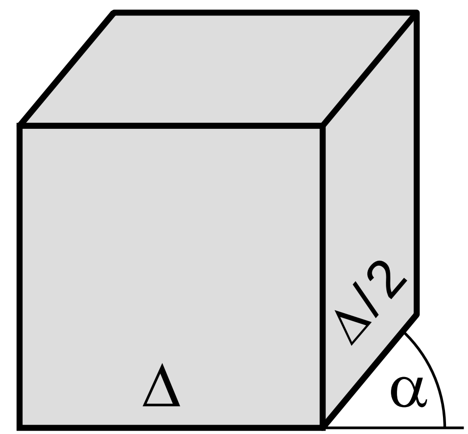
Beispiele für schiefe Projektion

$d = 1 \Rightarrow \beta = 45°$
$\alpha = 35°$



Kavalierprojektion

$d = 0.5 \Rightarrow \beta = 63.43°$
$\alpha = 50°$



Kabinettprojektion