Computergrafik SS 2008

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Kapitel 13: 3D-Transformationen

Einsatzgebiet

- Platzierung von Objekten in der Szene
- Berechnung der Projektion

Translation

$$(x', y', z') := (x + t_x, y + t_y, z + t_z)$$

$$T(t_x, t_y, t_z) = \begin{pmatrix} 1 & 0 & 0 & t_x \\ 0 & 1 & 0 & t_y \\ 0 & 0 & 1 & t_z \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

Skalierung

Fixpunkt im Ursprung:

$$(x', y', z') := (x \cdot s_x, y \cdot s_y, z \cdot s_z)$$

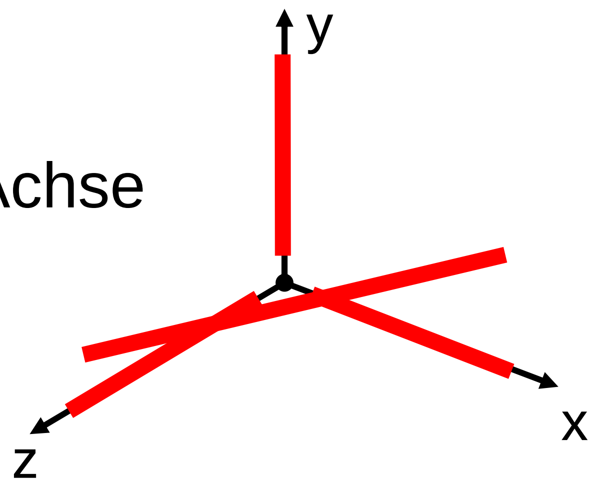
$$S(s_x, s_y, s_z) = \begin{pmatrix} s_x & 0 & 0 & 0 \\ 0 & s_y & 0 & 0 \\ 0 & 0 & s_z & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

Fixpunkt bei Z_x, Z_y, Z_z:

$$T(Z_x, Z_y, Z_z) \cdot S(s_x, s_y, s_z) \cdot T(-Z_x, -Z_y, -Z_z)$$

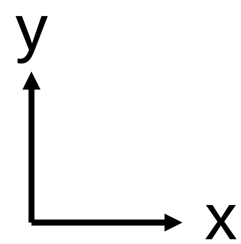
Rotation

- um z-Achse
- um x-Achse
- um y-Achse
- um beliebige Achse



Rotation um z-Achse

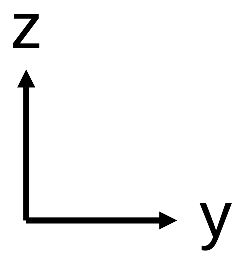
$$
\begin{aligned}
x' &:= x \cdot \cos(\delta) - y \cdot \sin(\delta) \\
y' &:= x \cdot \sin(\delta) + y \cdot \cos(\delta) \\
z' &:= z
\end{aligned}
$$



$$
R_z(\delta) = \begin{pmatrix} \cos(\delta) & -\sin(\delta) & 0 & 0 \\ \sin(\delta) & \cos(\delta) & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}
$$

Rotation um x-Achse

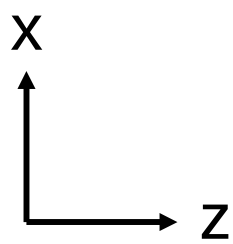
$$
\begin{aligned}
x' &:= x \\
y' &:= y \cdot \cos(\delta) - z \cdot \sin(\delta) \\
z' &:= y \cdot \sin(\delta) + z \cdot \cos(\delta)
\end{aligned}
$$



$$
R_x(\delta) = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos(\delta) & -\sin(\delta) & 0 \\ 0 & \sin(\delta) & \cos(\delta) & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}
$$

Rotation um y-Achse

$$
\begin{aligned}
x' &:= z \cdot \sin(\delta) + x \cdot \cos(\delta) \\
y' &:= y \\
z' &:= z \cdot \cos(\delta) - x \cdot \sin(\delta)
\end{aligned}
$$

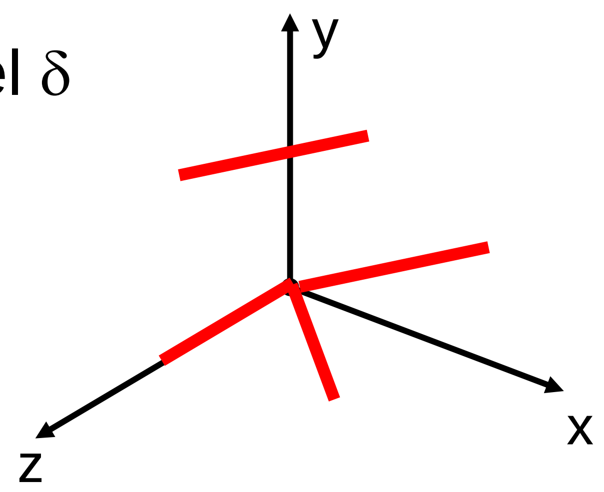


$$
R_y(\delta) = \begin{pmatrix} \cos(\delta) & 0 & \sin(\delta) & 0 \\ 0 & 1 & 0 & 0 \\ -\sin(\delta) & 0 & \cos(\delta) & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}
$$

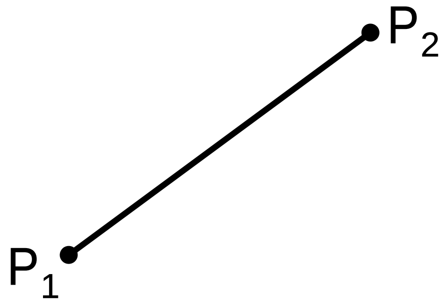
Rotation um beliebige Achse

Punkt P Drehwinkel δ Drehachse P_2-P_1

1. Translation in den Ursprung
2. Rotation um die x-Achse in die xz-Ebene
3. Rotation um die y-Achse in die z-Achse
4. Rotation um die z-Achse mit Winkel δ
5. Inversion von Schritt 3
6. Inversion von Schritt 2
7. Inversion von Schritt 1



Drehachse



$$\vec{v} = P_2 - P_1 = \begin{pmatrix} x_2 - x_1 \\ y_2 - y_1 \\ z_2 - z_1 \end{pmatrix}$$

$$|\vec{v}| = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}$$

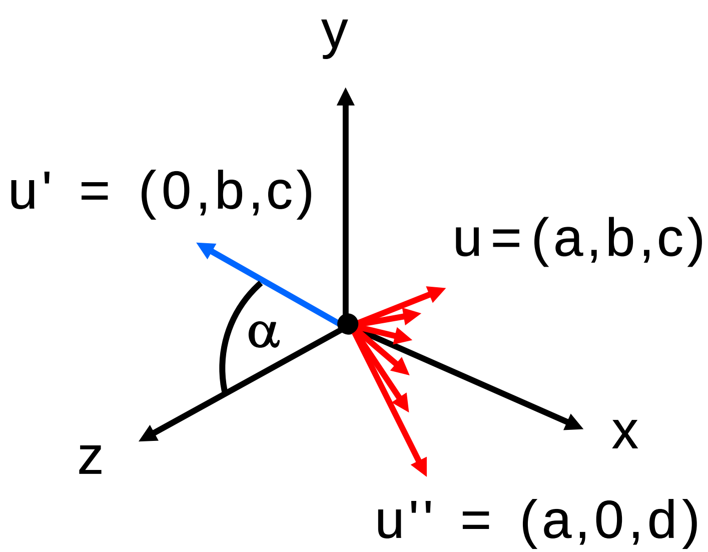
$$\vec{u} = \frac{\vec{v}}{|\vec{v}|} = \begin{pmatrix} a \\ b \\ c \end{pmatrix} \qquad |\vec{u}| = 1$$

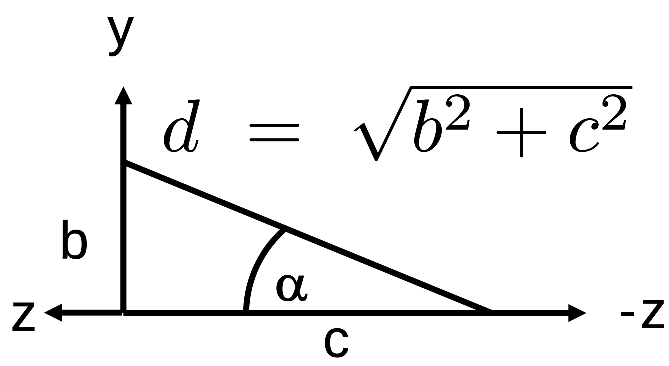
$$a = \frac{x_2 - x_1}{|\vec{v}|} \quad b = \frac{y_2 - y_1}{|\vec{v}|} \quad c = \frac{z_2 - z_1}{|\vec{v}|}$$

1.) Translation in den Ursprung

$$T(-x_1, -y_1, -z_1)$$

2.) Rotation um die x-Achse in die xz-Ebene



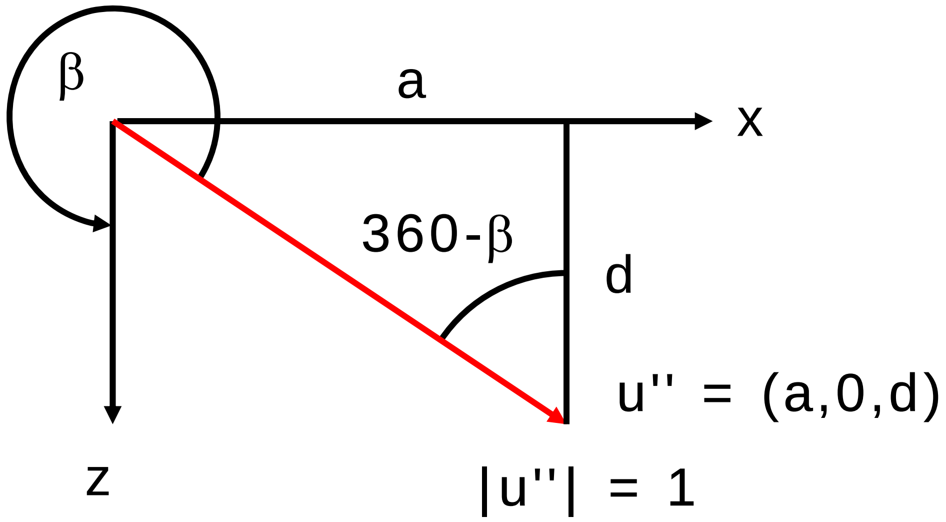


$$d = \sqrt{b^2 + c^2}$$

$\cos(\alpha) = c/d$

$\sin(\alpha) = b/d$

3.) Rotation um die y-Achse in die z-Achse



$$\Rightarrow \cos(\beta) = \cos(360^\circ - \beta) = d$$

$$\Rightarrow \sin(\beta) = -\sin(360^\circ - \beta) = -a$$

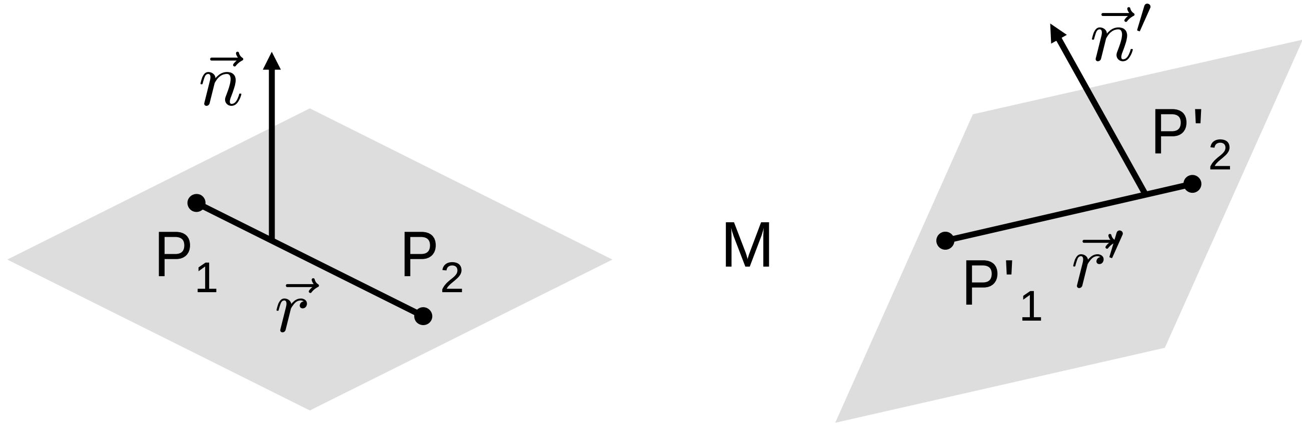
4.) Rotation um die z-Achse

$$R_z(\delta) = \begin{pmatrix} \cos(\delta) & -\sin(\delta) & 0 & 0 \\ \sin(\delta) & \cos(\delta) & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

1. - 7.) Gesamttransformation

$$
\begin{aligned}
R(\vec{v}, \delta) \;=\; & T(-P_1) \\
& R_x(\alpha) \cdot \\
& R_y(\beta) \cdot \\
& R_z(\delta) \cdot \\
& R_y^{-1}(\beta) \cdot \\
& R_x^{-1}(\alpha) \cdot \\
& T(P_1) \cdot
\end{aligned}
$$

Transformation einer Normalen



$$\vec{r} = P_2 - P_1 \qquad P_1' = M \cdot P_1 \quad P_2' = M \cdot P_2$$

$$\vec{r}' = P_2' - P_1'$$

$$= M \cdot P_2 - M \cdot P_1 = M \cdot (P_2 - P_1) = M \cdot \vec{r}$$

$$\vec{n}^T \cdot r = 0 \qquad M \cdot \vec{r} = \vec{r}' \qquad \vec{n}'^T \cdot \vec{r}' = 0$$

Umformungen

$$\vec{n}^T \cdot r = 0$$

$$\vec{n}^T \cdot M^{-1} \cdot M \cdot r = 0$$

$$((M^{-1})^T \cdot \vec{n})^T \cdot \underbrace{M \cdot r}_{\vec{r}'} = 0$$

$$\vec{n}'^T \cdot \vec{r}' = 0$$

$$((M^{-1})^T \cdot \vec{n})^T = \vec{n}'^T$$

$$((M^{-1})^T \cdot \vec{n}) = \vec{n}'$$

$\Rightarrow$ transformiere den Normalenvektor
mit der transponierten Inversen !

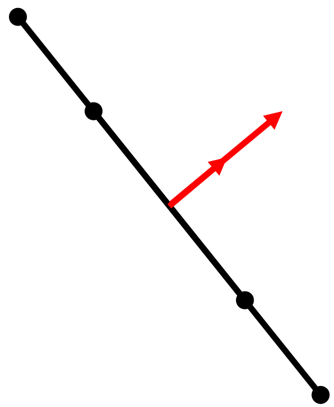
Skalierung einer Normalen

$$\begin{matrix} 0.8 \\ 0.6 \\ 0.0 \end{matrix}$$

skaliere um Faktor 2

M

$$\begin{matrix} 2.0 & 0.0 & 0.0 \\ 0.0 & 2.0 & 0.0 \\ 0.0 & 0.0 & 1.0 \end{matrix}$$



$$\begin{matrix} 0.4 \\ 0.3 \\ 0.0 \end{matrix} \qquad \begin{matrix} 0.8 \\ 0.6 \\ 0.0 \end{matrix}$$

M^{-1}

$$\begin{matrix} 0.5 & 0.0 & 0.0 \\ 0.0 & 0.5 & 0.0 \\ 0.0 & 0.0 & 1.0 \end{matrix}$$

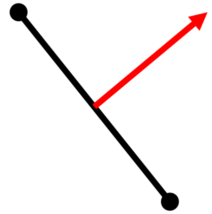
$(M^{-1})^T$

$$\begin{matrix} 0.5 & 0.0 & 0.0 \\ 0.0 & 0.5 & 0.0 \\ 0.0 & 0.0 & 1.0 \end{matrix}$$

Rotation einer Normalen

```
0.8
0.6
0.0
```

drehe um 25°



```
0.48
0.88
0.00
```

M

```
0.91 -0.42 0.0
0.42  0.91 0.0
0.0   0.0  1.0
```

M^{-1}

```
 0.91 0.42 0.0
-0.42 0.91 0.0
 0.0  0.0  1.0
```

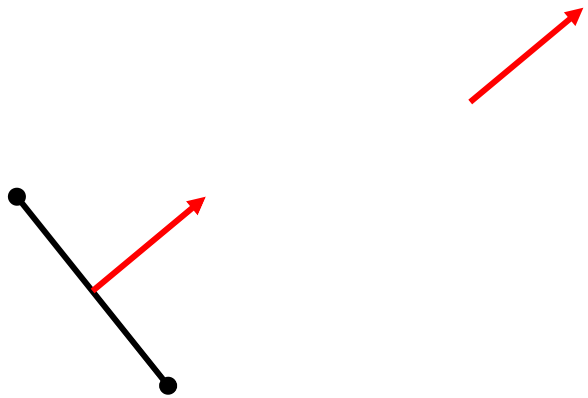
$(M^{-1})^T$

```
0.91 -0.42 0.0
0.42  0.91 0.0
0.0   0.0  1.0
```

Translation einer Normalen

verschiebe um (5,2)

$$\begin{pmatrix} 0.8 \\ 0.6 \\ 0.0 \end{pmatrix}$$



$$\begin{pmatrix} 0.8 \\ 0.6 \\ 0.0 \end{pmatrix}$$

$$M = \begin{pmatrix} 1 & 0 & 5 \\ 0 & 1 & 2 \\ 0 & 0 & 1 \end{pmatrix} \qquad M^{-1} = \begin{pmatrix} 1 & 0 & -5 \\ 0 & 1 & -2 \\ 0 & 0 & 1 \end{pmatrix} \qquad (M^{-1})^T = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ -5 & -2 & 1 \end{pmatrix}$$