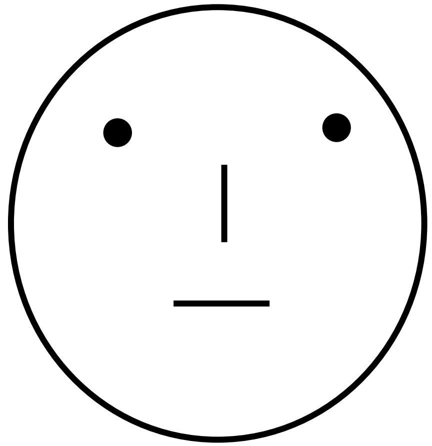
# Computergrafik SS 2010
Oliver Vornberger

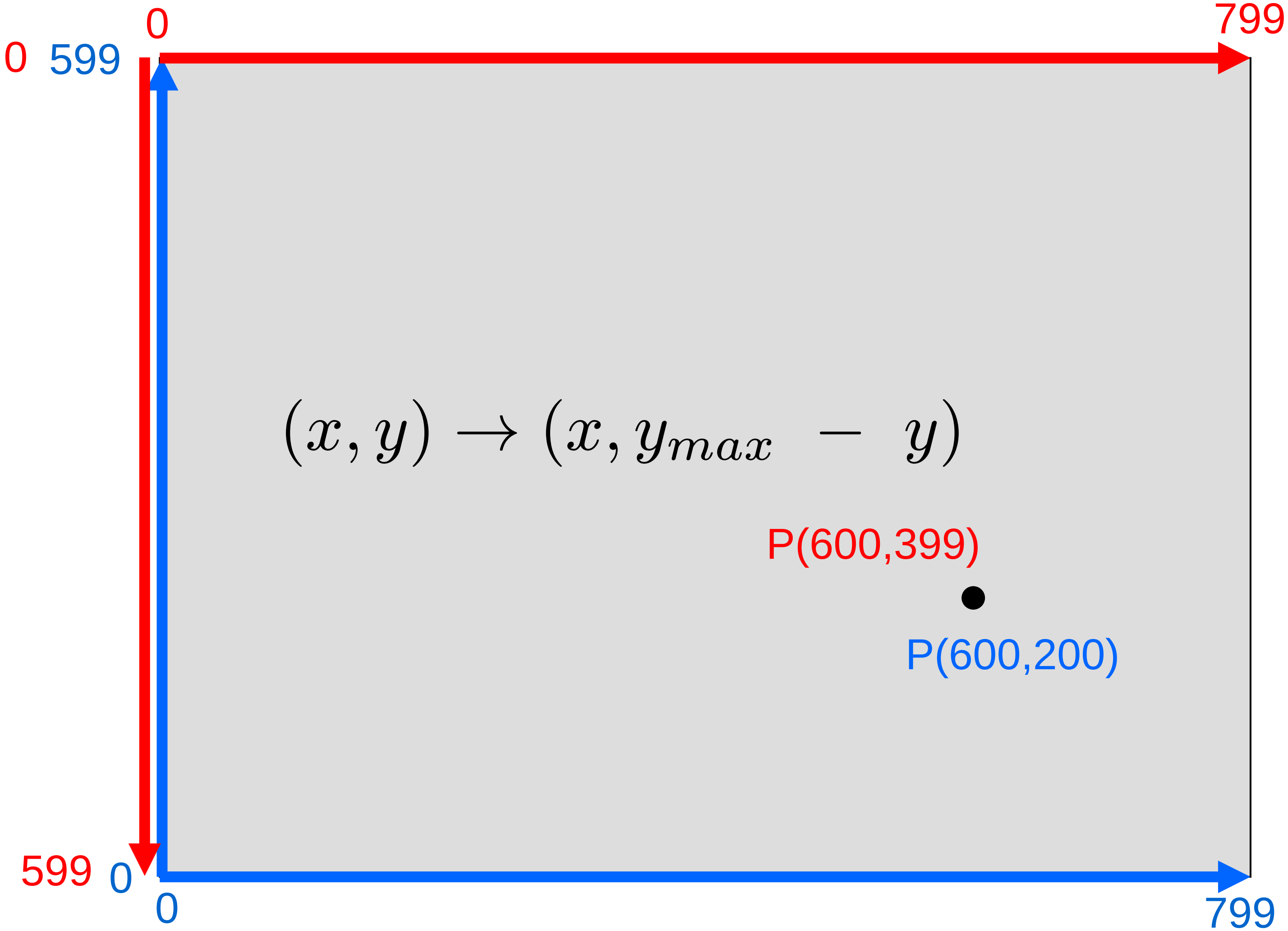
# Kapitel 3:
2D-Grundlagen

# Punkt, Punkt, Komma, Strich, ...



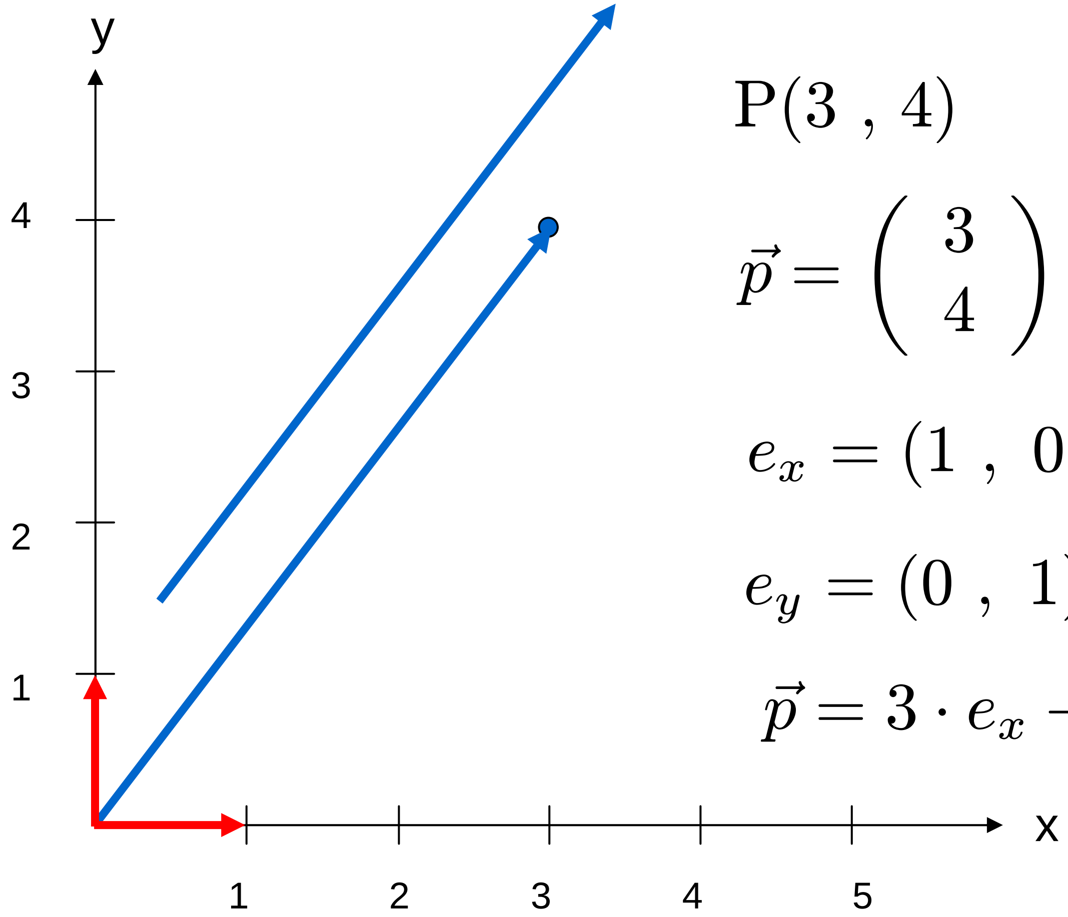
... fertig ist das Mondgesicht !

# Koordinatensysteme




$$(x, y) \rightarrow (x, y_{max} - y)$$

# Punkt + Vektor




$$P(3\ ,\ 4)$$

$$\vec{p} = \begin{pmatrix} 3 \\ 4 \end{pmatrix} = (3\ ,\ \ 4)^T$$

$$e_x = (1\ ,\ \ 0)^T$$

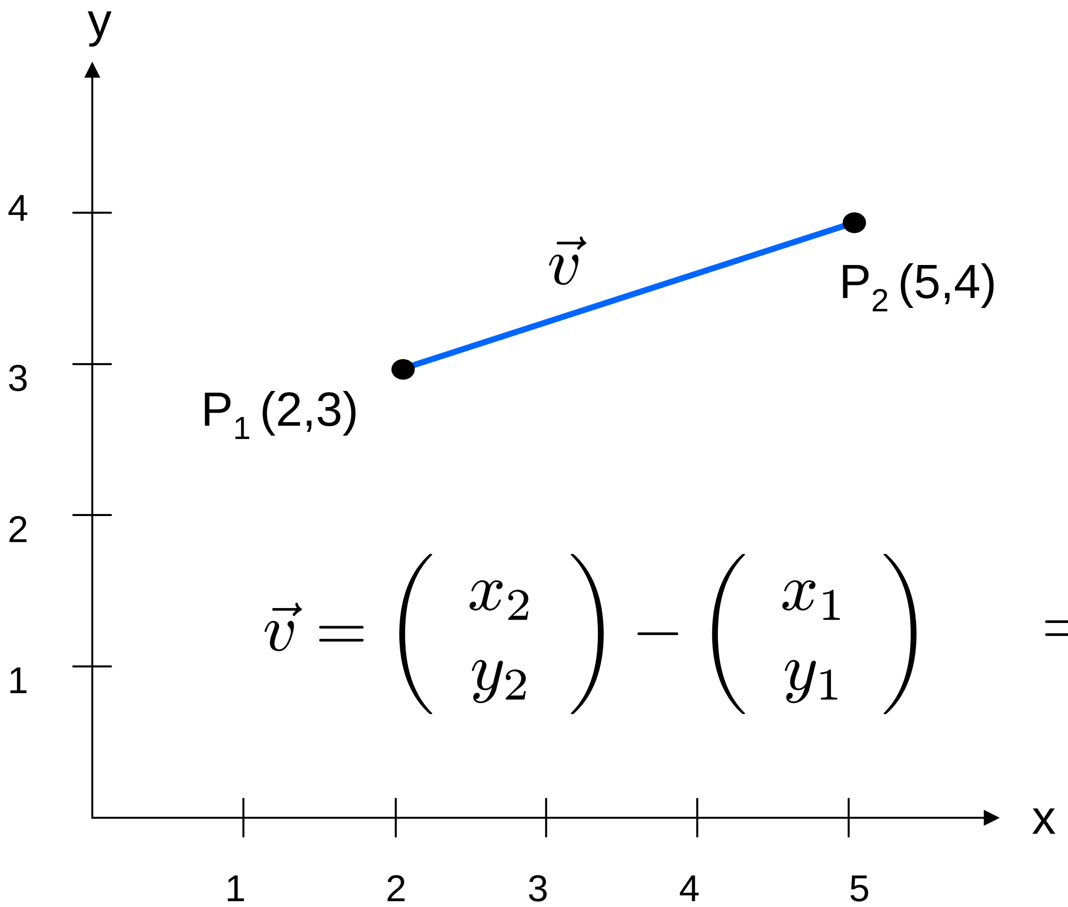
$$e_y = (0\ ,\ \ 1)^T$$

$$\vec{p} = 3 \cdot e_x + 4 \cdot e_y$$

# setPixel(int x, int y)

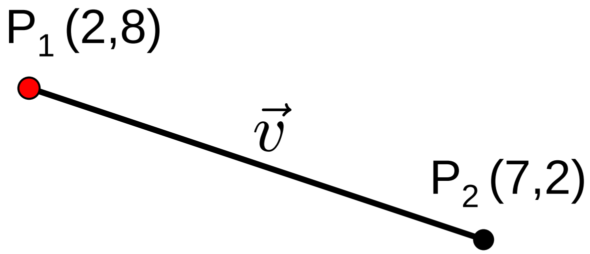
```
setPixel(3,4);

setPixel((int)(x+0.5),(int)(y+0.5));
```

# Linie




$$\vec{v} = \begin{pmatrix} x_2 \\ y_2 \end{pmatrix} - \begin{pmatrix} x_1 \\ y_1 \end{pmatrix} = \begin{pmatrix} x_2 - x_1 \\ y_2 - y_1 \end{pmatrix}$$

# Parametrisierte Gradengleichung




$$g : \vec{u} = \vec{p_1} + r \cdot \vec{v};\ \ r \in \mathbb{R}$$

$$l : \vec{u} = \vec{p_1} + r \cdot \vec{v};\ \ r \in [0; 1]$$

**1.0000**

$$P = (1 - t) \cdot P_1 + t \cdot P_2$$

$$d = \|\overline{P_1 P_2}\| \quad = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$step = \frac{1}{\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}}$$

# VectorLine

```
int x1,y1,x2,y2,x,y,dx,dy;
double r, step;

dy = y2-y1;
dx = x2-x1;

step = 1.0/Math.sqrt(dx*dx+dy*dy);
for (r=0.0; r <= 1; r=r+step) {
  x = (int)(x1+r*dx+0.5);
  y = (int)(y1+r*dy+0.5);
  setPixel(x,y);
}
```

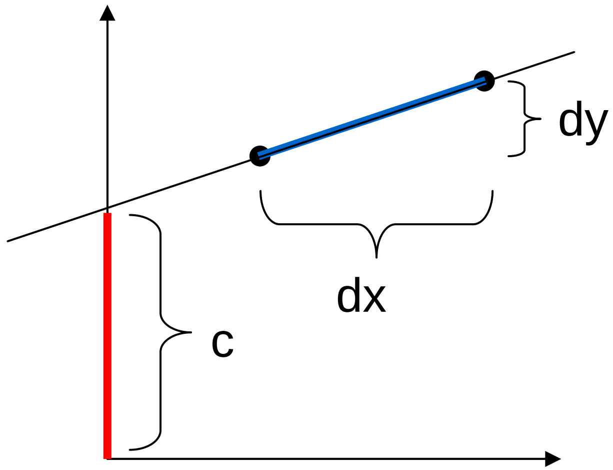
# Gradengleichung als Funktion

$$y = f(x) = s \cdot x + c$$

$$s = \frac{y_2 - y_1}{x_2 - x_1}$$

$$\frac{y_1 - c}{x_1 - 0} = \frac{y_2 - y_1}{x_2 - x_1}$$

$$c = \frac{y_1 \cdot x_2 - y_2 \cdot x_1}{x_2 - x_1}$$



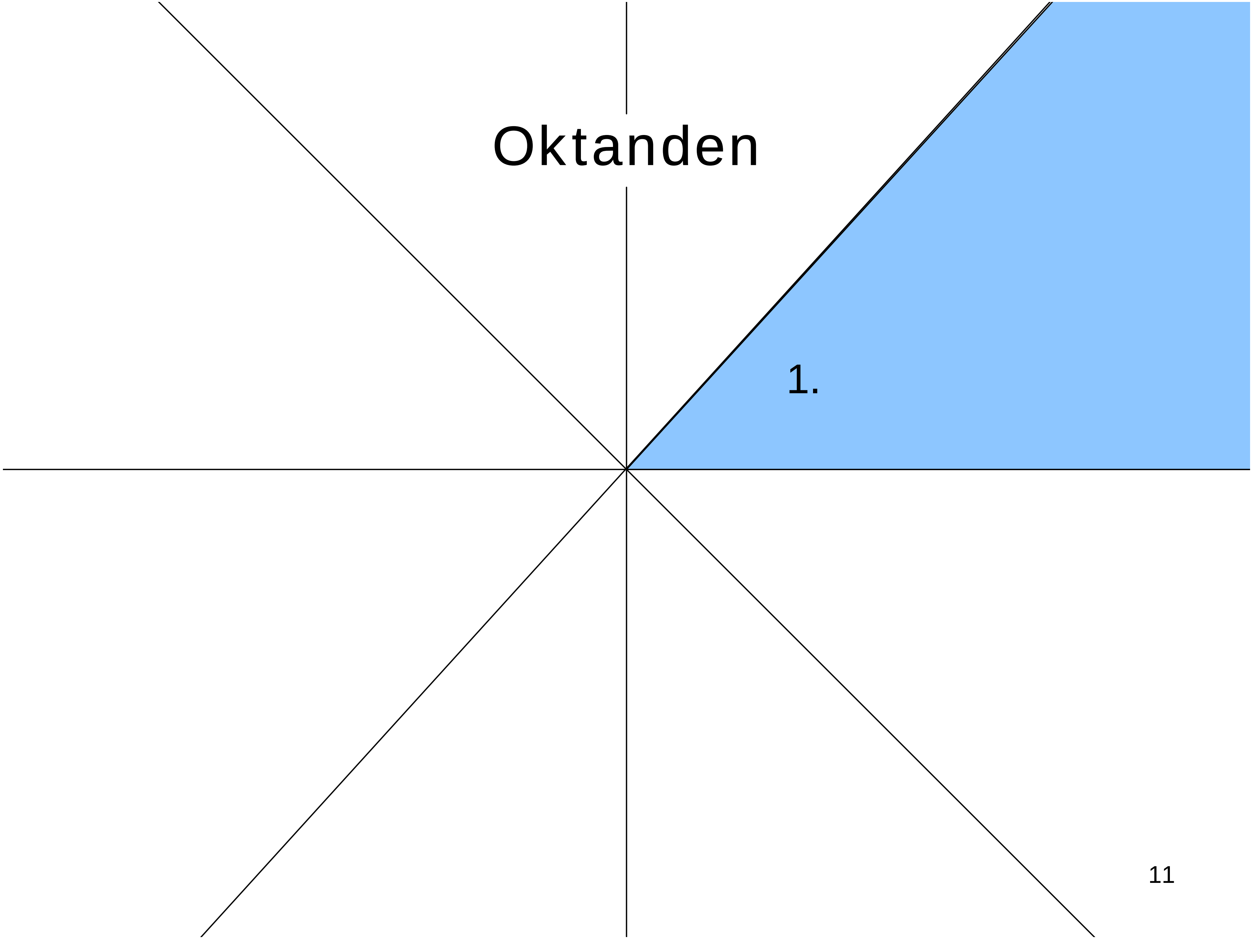

$$y = \frac{y_2 - y_1}{x_2 - x_1} \cdot x + \frac{x_2 \cdot y_1 - x_1 \cdot y_2}{x_2 - x_1}$$

# StraightLine

von links nach rechts

```
s = (double)(y2-y1)/(double)(x2-x1);
c = (double)(x2*y1-x1*y2)/(double)(x2-x1);

for (x=x1; x <= x2; x++) {
  y = (int)(s*x+c+0.5);
  setPixel(x,y);
}
```

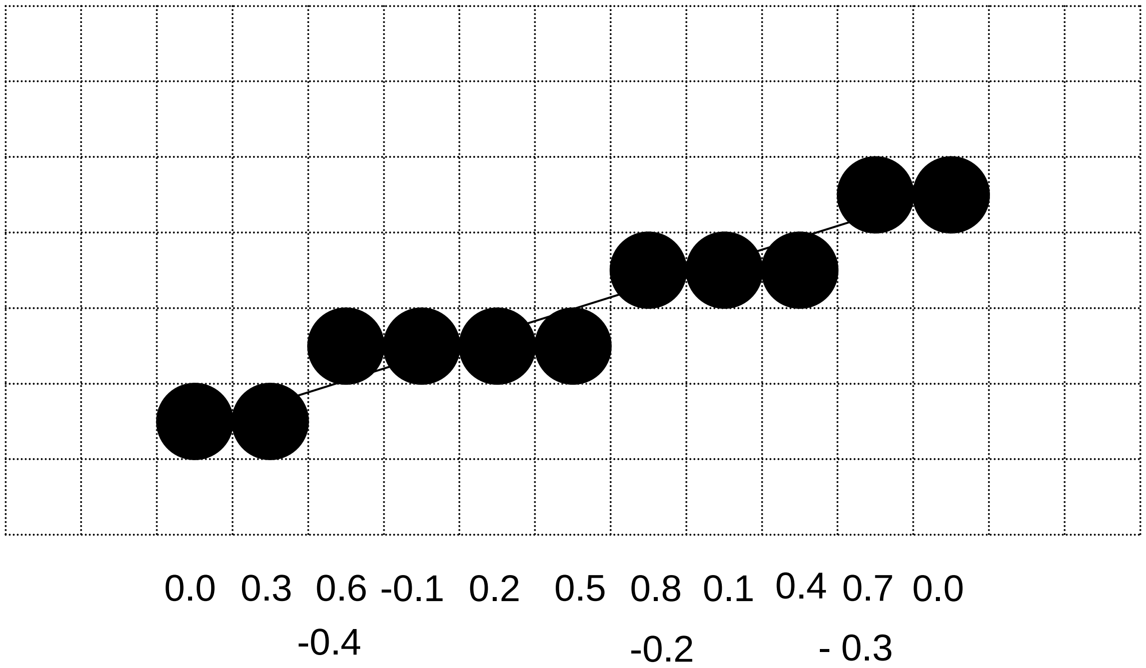
# Oktanden



# Bresenham

Steigung $s = \Delta y / \Delta x = 3/10 = 0.3$

Fehler $error = y_{ideal} - y_{real}$



# BresenhamLine, die 1.

```
dy = y2-y1; dx = x2-x1;
s = (double)dy/(double)dx;
error = 0.0;
x = x1;
y = y1;
while (x <= x2){
  setPixel(x,y);
  x++;
  error = error + s;
  if (error > 0.5) {
    y++;
    error = error – 1.0;
  }
}
```

# Integer-Arithmetik

Mache Steigung + Fehler ganzzahlig:

$$dx := x_2 - x_1$$

$$dy := y_2 - y_1$$

$$s_{neu} = s_{alt} \cdot 2dx = \frac{dy}{dx} \cdot 2dx = 2dy$$

# BresenhamLine, die 2.

```
dy = y2-y1; dx = x2-x1;
s = (double)dy/(double)dx;   delta = 2*dy
error = 0.0;
x = x1;
y = y1;
while (x <= x2){
  setPixel(x,y);
  x++;
  error = error + s;              delta
  if (error > 0.5) {              dx
    y++;
    error = error - 1.0;          2*dx
  }
}
```

multipliziere Steigung mit 2dx

# Vergleich mit 0

- vergleiche **error** mit 0,
  d.h. verschiebe **error** um (x2-x1) nach unten
- verwende **schritt** für -2*dx

# BresenhamLine, die 3.

```
dy = y2-y1; dx = x2-x1;
delta = 2*dy;
error = 0.0;                       -dx
x = x1;                            schritt= -2*dx
y = y1;
while (x <= x2){
  setPixel(x,y);
  x++;
  error = error + delta;
  if (error > dx) {                0
    y++;
    error = error - 2*dx;          + schritt
  }
}
```

Verschiebe error um 2dx nach unten

# BresenhamLine

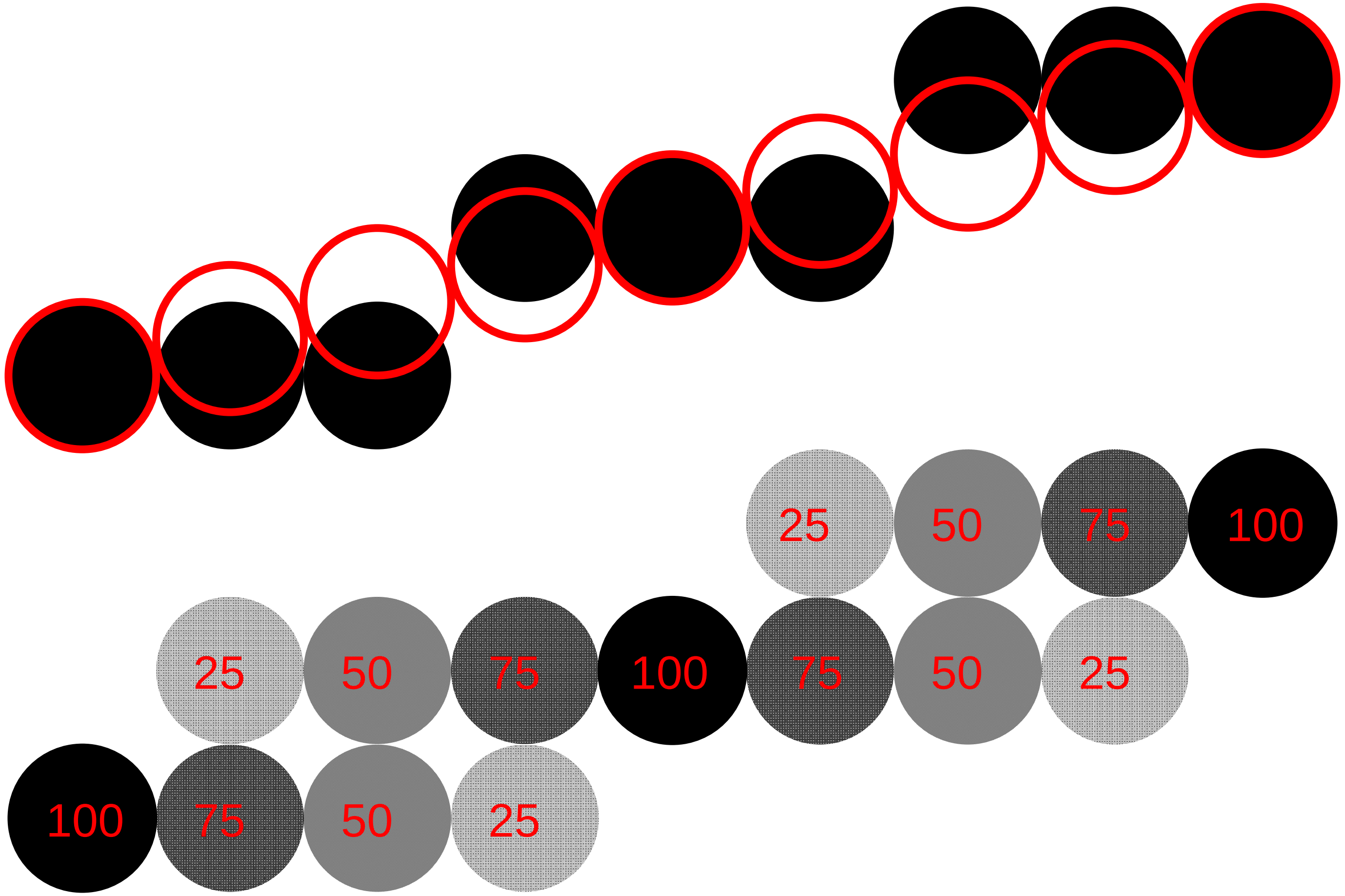
alle 8 Oktanden durch Fallunterscheidung abhandeln:

~cg/2010/skript/Sources/drawBresenhamLine.jav.html

Java-Applet:

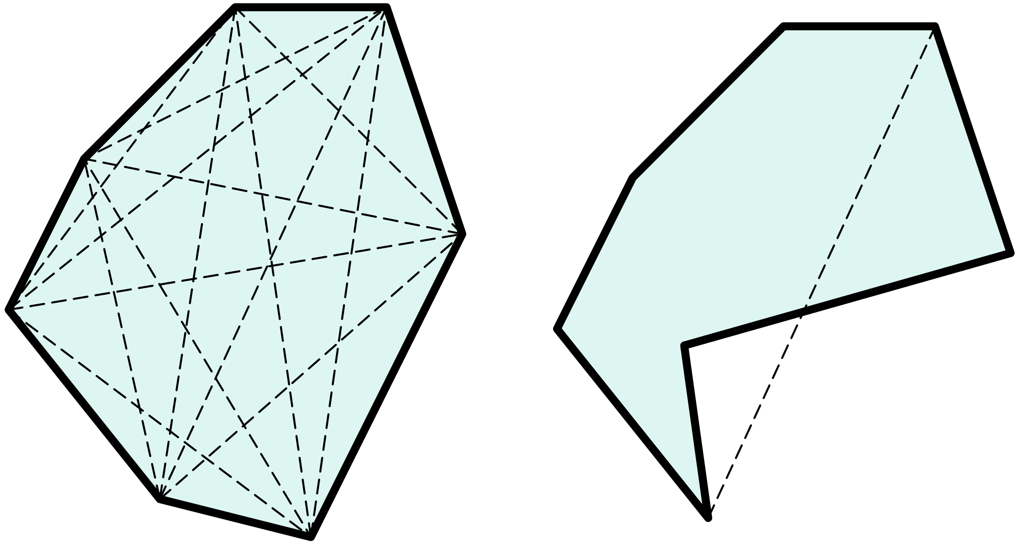
~cg/2010/skript/Applets/2D-basic/App.html

# Antialiasing



# Antialiasing in Adobe Photoshop

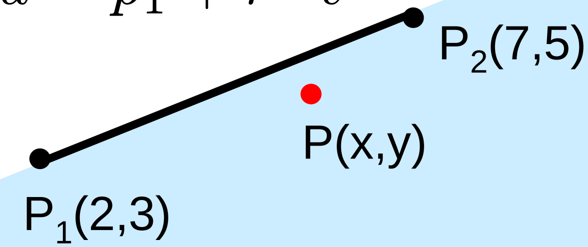
# Polygon



# Punkt versus Gerade

$$\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 2 \\ 3 \end{pmatrix} + r \cdot \begin{pmatrix} 7-2 \\ 5-3 \end{pmatrix}$$

$$\vec{u} = \vec{p_1} + r \cdot \vec{v}$$



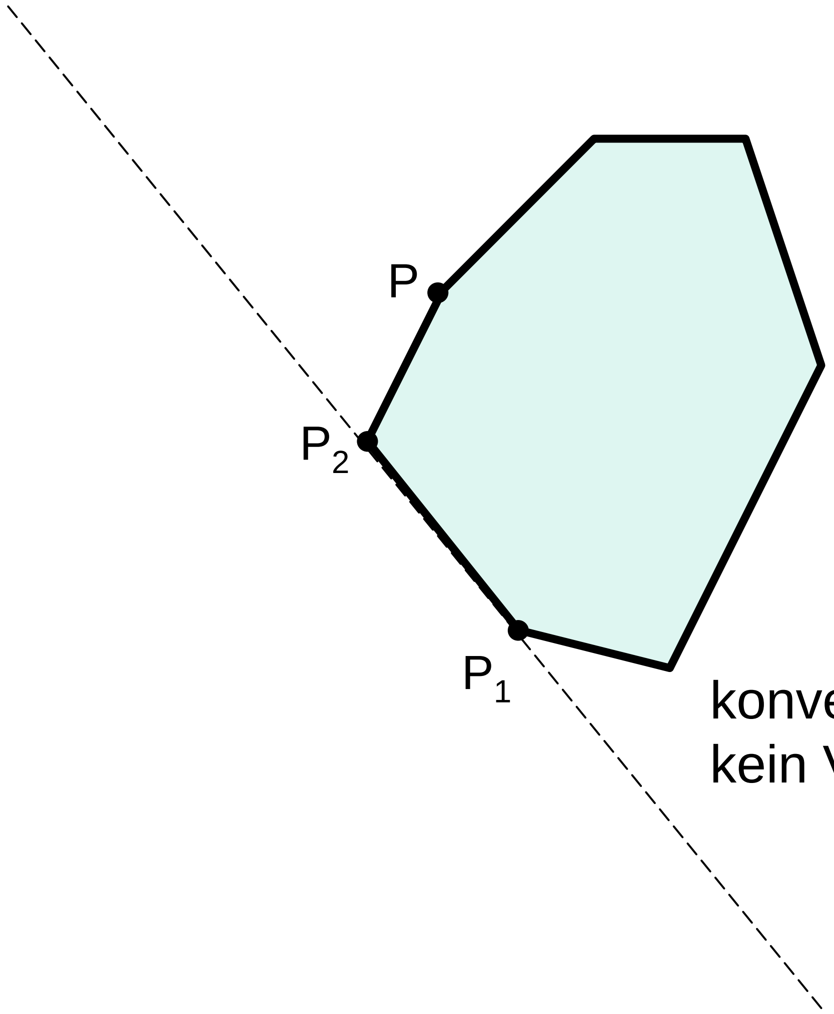

$$\begin{aligned} x &= 2 + 5r \\ y &= 3 + 2r \\ 2x &= 4 + 10r \\ -5y &= -15 - 10r \\ 2x - 5y &= -11 \\ 2x - 5y + 11 &= 0 \end{aligned}$$

*F(x,y)* = *0* falls *P* auf der Geraden

> *0* falls *P* rechts von der Geraden

< *0* falls *P* links von der Geraden

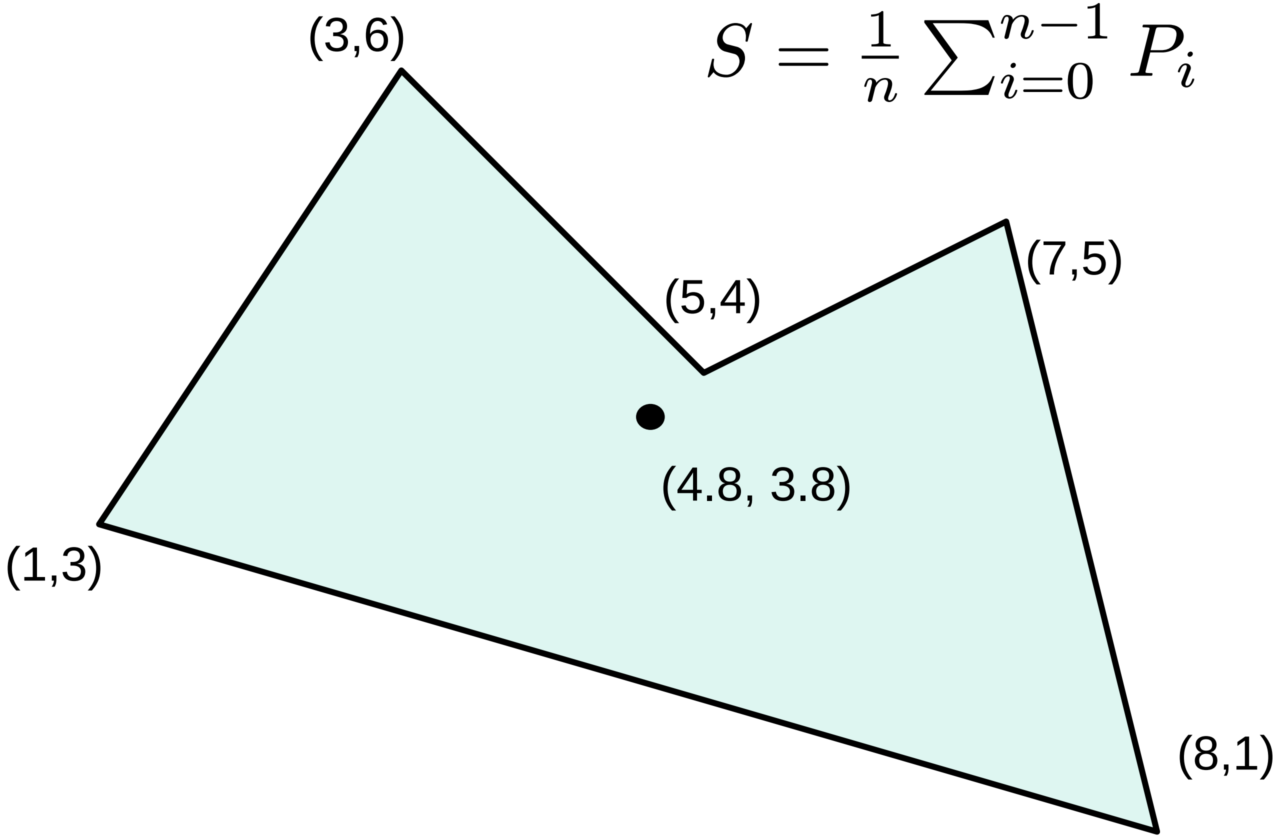
# Konvexitätstest nach Paul Bourke



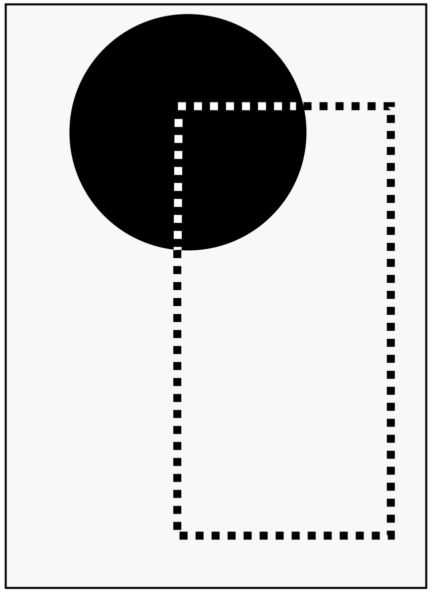

konvex, falls für alle P(x,y)
kein Vorzeichenwechsel bei F(x,y)

# Schwerpunkt

$$S = \frac{1}{n} \sum_{i=0}^{n-1} P_i$$



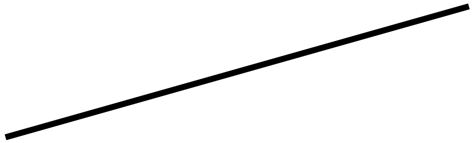
# Zeichnen und Löschen mit XOR



| | |
|---|---|
| Pixel: | 01101011 |
| Gummiband: | 11111111 |
| XOR ergibt: | 10010100 |
| Gummiband: | 11111111 |
| XOR ergibt: | 01101011 |

Beispiel für Gummiband:
~cg/2010/skript/Applets/2D-basic/App.html

# Algorithmen zum Zeichnen



Parametrisiert:

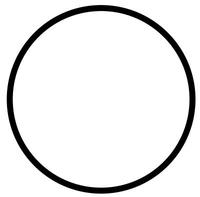
$x := f_1(t);\ y := f_2(t)$

Gradengleichung:

$y := f(x)$

Bresenham:

**x++; if (…) {y++; … }**



Parametrisiert:

$x = f_1(t);\ y := f_2(t)$

Kreisgleichung:

$y := f(x)$

Bresenham:

**x++; if (…) {y--; … }**

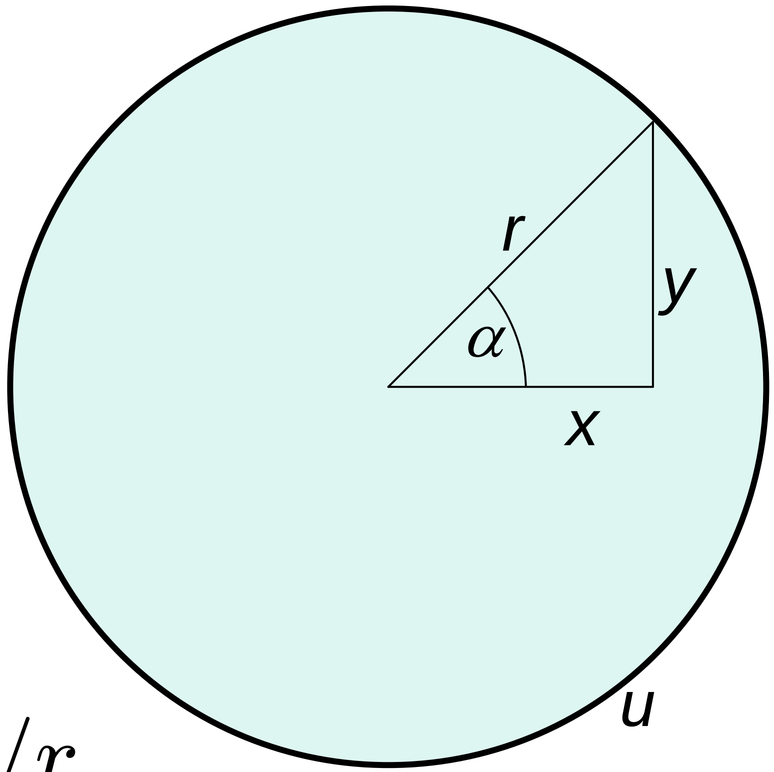
# Kreis um (0,0), parametrisiert

$$x = r \cdot \cos(\alpha)$$

$$y = r \cdot \sin(\alpha)$$

$$u = 2 \cdot \pi \cdot r$$

$$step = \frac{2 \cdot \pi}{2 \cdot \pi \cdot r} = 1/r$$



# TriCalcCircle

```
double step = 1.0/(double r);
double winkel;

for (winkel = 0.0;
     winkel < 2*Math.PI;
     winkel = winkel+step){

  setPixel((int) r*Math.sin(winkel)+0.5,
           (int) r*Math.cos(winkel)+0.5);
}
```

# TriTableCircle

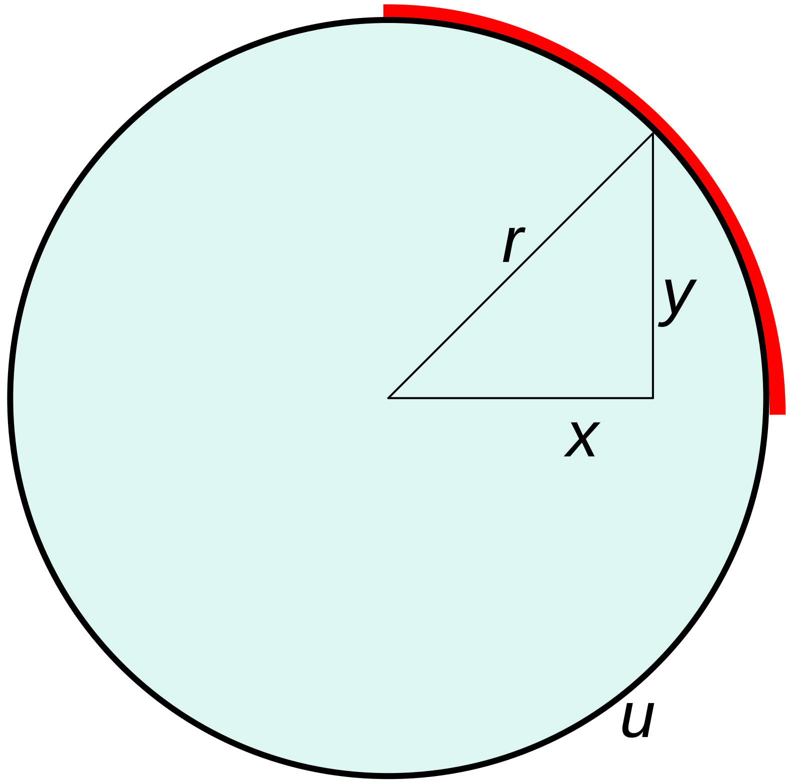
```
// Tabellen sin + cos seien berechnet
// für ganzzahlige Winkel von 0..360

int winkel;

for (winkel = 0;
     winkel < 360;
     winkel++){

  setPixel((int) r*sin[winkel] + 0.5,
           (int) r*cos[winkel] + 0.5);
}
```

Problem: konstante Zahl von Kreispunkten !

# Kreis als Funktion im 1. Quadranden

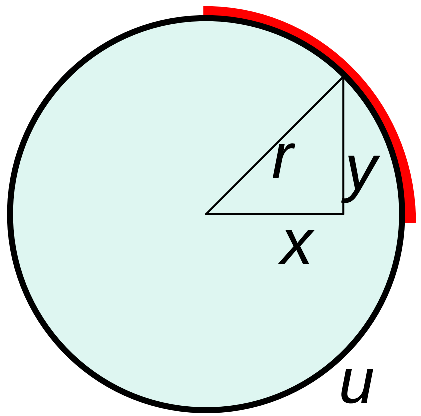
$$x^2 + y^2 = r^2$$

$$y = \sqrt{r^2 - x^2}$$



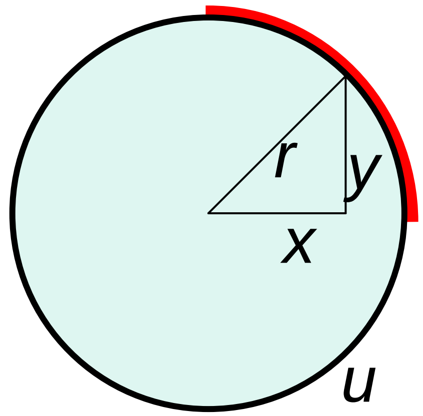
# PythagorasCircle, die 1.

$$y = \sqrt{r^2 - x^2}$$




```
for (x=0; x <=r; x++){
  y = (int) Math.sqrt(r*r-x*x);
  setPixel(x,y);
}
```

# PythagorasCircle, die 2.

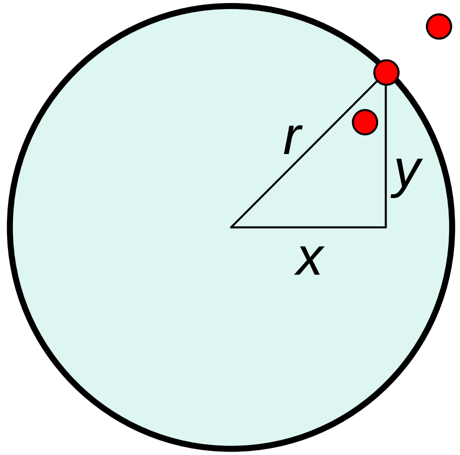



$$y = \sqrt{r^2 - x^2}$$

$$w = r^2,\ r^2 - 1,\ r^2 - 4,\ r^2 - 9,\ r^2 - 16, \ldots$$

```
d = 1;
w = r*r;
for (x=0; x <= r; x++) ){
  y = (int) Math.sqrt(w);
  setPixel(x,y);
  w = w-d
  d = d+2;
}
```

# Punkt versus Kreis




$$x^2 + y^2 = r^2$$

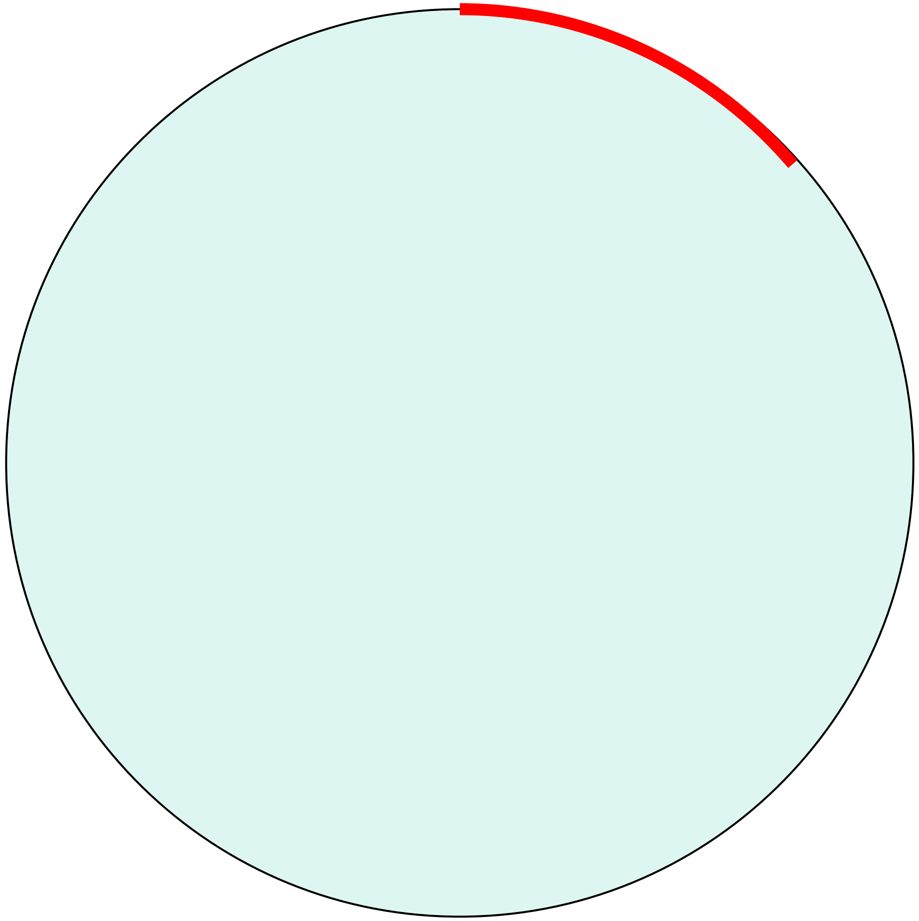
$$F(x,y) = x^2 + y^2 - r^2$$

$F(x,y)$ = 0 für $(x,y)$ auf dem Kreis

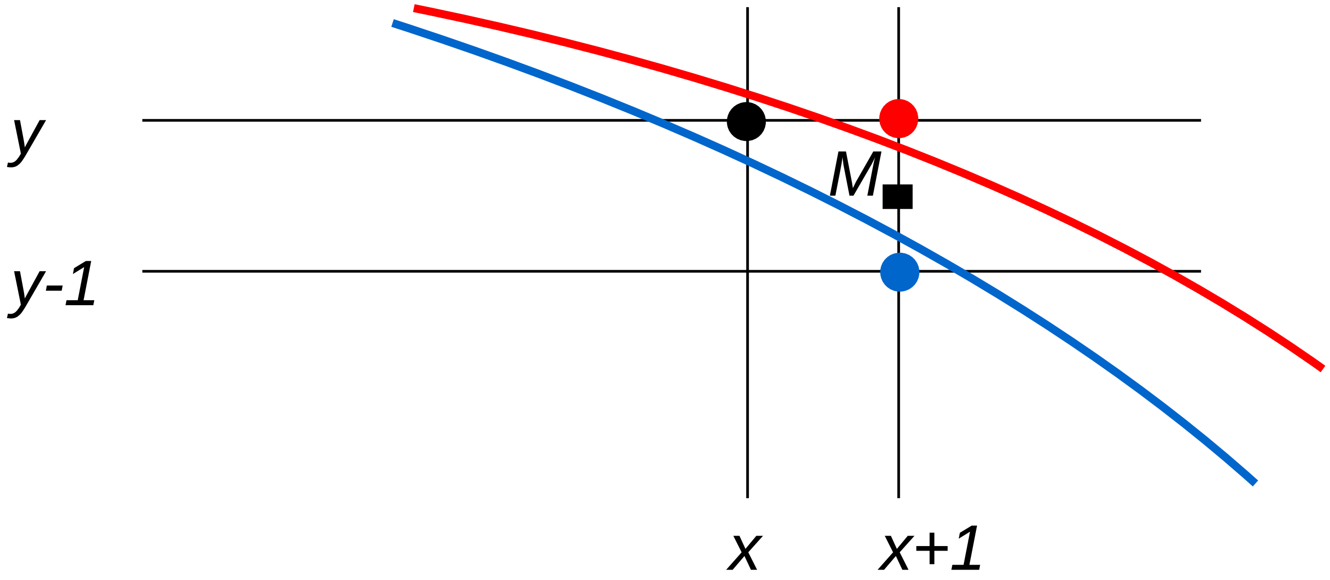
< 0 für $(x,y)$ innerhalb des Kreises

> 0 für $(x,y)$ außerhalb des Kreises

# Kreis im 2. Oktanden



# Entscheidungsvariable $\Delta$




$\Delta = F(x+1, y-½)$

$\Delta < 0 \Rightarrow M$ liegt innerhalb $\Rightarrow$ wähle $(x+1,y)$

$\Delta \geq 0 \Rightarrow M$ liegt außerhalb $\Rightarrow$ wähle $(x+1,y-1)$

# Berechnung von $\Delta$

$\Delta = F(x+1, y-½) = (x+1)^2+(y-½)^2-r^2$

$\Delta < 0 \Rightarrow$

$\Delta' = F(x+2, y-½) = (x+2)^2+(y-½)^2-r^2 =$

$\Delta+2x+3$

$\Delta \geq 0 \Rightarrow$

$\Delta' = F(x+2, y-3/2) = (x+2)^2+(y-3/2)^2-r^2 =$

$\Delta+2x-2y+5$

Startwert $\Delta = F(1, r-½) = 1^2+(r-½)^2 - r^2 =$

$5/4-r$

# BresenhamCircle, die 1.

```
x = 0;
y = r;
delta = 5.0/4.0 - r;
while (y >= x) {
  setPixel(x,y);
  if (delta < 0.0) {
    delta = delta + 2*x + 3.0;
    x++;
  else {
    delta = delta + 2*x - 2*y + 5.0;
    x++;
    y--;
  }
}
```

# Substitutionen

$d := delta - \frac{1}{4}$

$dx := 2x+3$

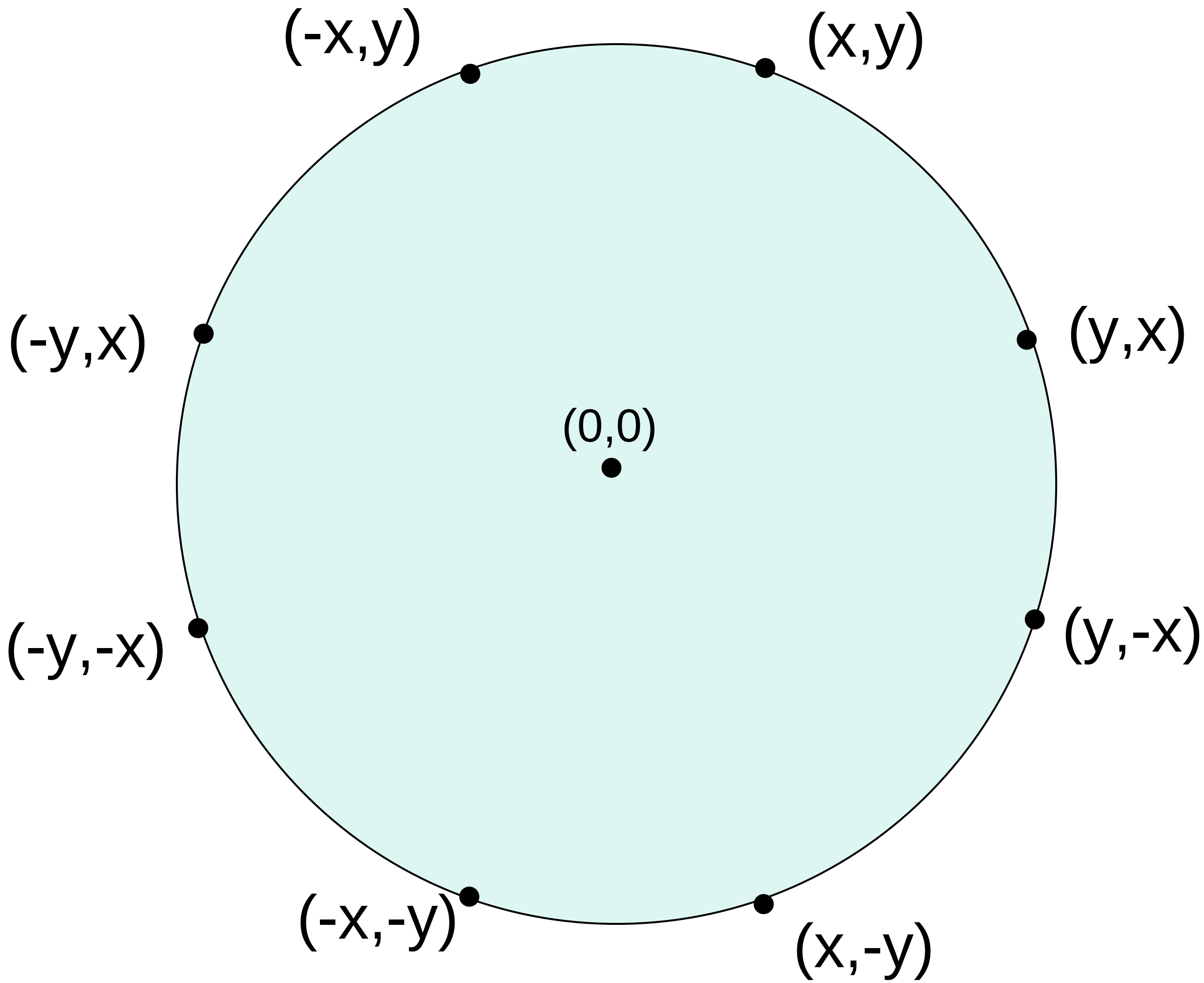
$dxy := 2x - 2y + 5$

# BresenhamCircle, die 2.

```
x = 0;
y = r;
delta = 5.0/4.0 - r;                    d = 1 - r;
                                        dx = 3;
                                        dxy = -2*r + 5;
while (y >= x) {
  setPixel(x,y);
  if (delta < 0.0) {                    (d <= 0.0)
    delta = delta + 2*x + 3.0;          d = d + dx;
    x++;                                dx = dx + 2;
  else {                                dxy = dxy + 2;
    delta = delta+2*x-2*y+5.0;          d   = d + dxy;
    x++;                                dx  = dx + 2;
    y--;                                dxy = dxy + 4;
  }
}
```

d:=delta-1/4 dx:=2x+3 dxy:= 2x-2y+5

# Oktanden-Symmetrie



# BresenhamCircle, die 3.

```
x=0; y=r; d=1-r; x=3; dx=3; dxy=-2*r+5;

while (y>=x){

   setPixel(+x,+y);
   setPixel(+y,+x);
   setPixel(+y,-x);
   setPixel(+x,-y);
   setPixel(-x,-y);
   setPixel(-y,-x);
   setPixel(-y,+x);
   setPixel(-x,+y);

  if (d<0) {d=d+dx;  dx=dx+2; dxy=dxy+2; x++;}
  else     {d=d+dxy; dx=dx+2; dxy=dxy+4; x++; y--;}
}
```

# BresenhamCircle

Zahl der erzeugten Punkte = $4 \cdot \sqrt{2} \cdot r$

= 10% unterhalb von $2 \cdot \pi \cdot r$

Kreis mit Radius r um Mittelpunkt (x,y):

~cg/2010/skript/Sources/drawBresenhamCircle.jav

Java-Applet:

~cg/2010/skript/Applets/2D-basic/App.html

Java-Applet für Performance-Messung

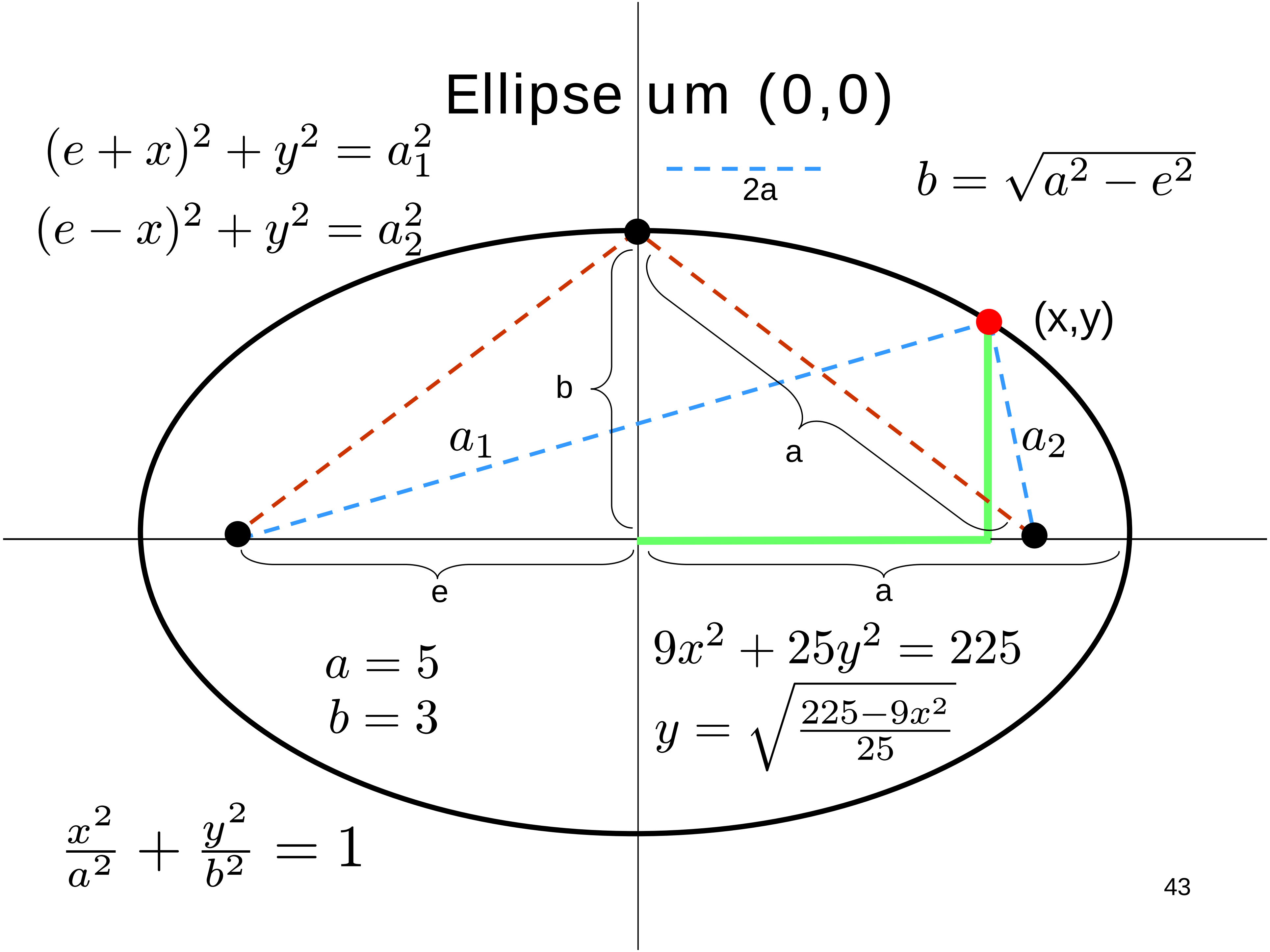
~cg/2010/skript/Applets/circle/App.html

# Ellipse um (0,0)

$$(e+x)^2 + y^2 = a_1^2$$

$$(e-x)^2 + y^2 = a_2^2$$

$$b = \sqrt{a^2 - e^2}$$



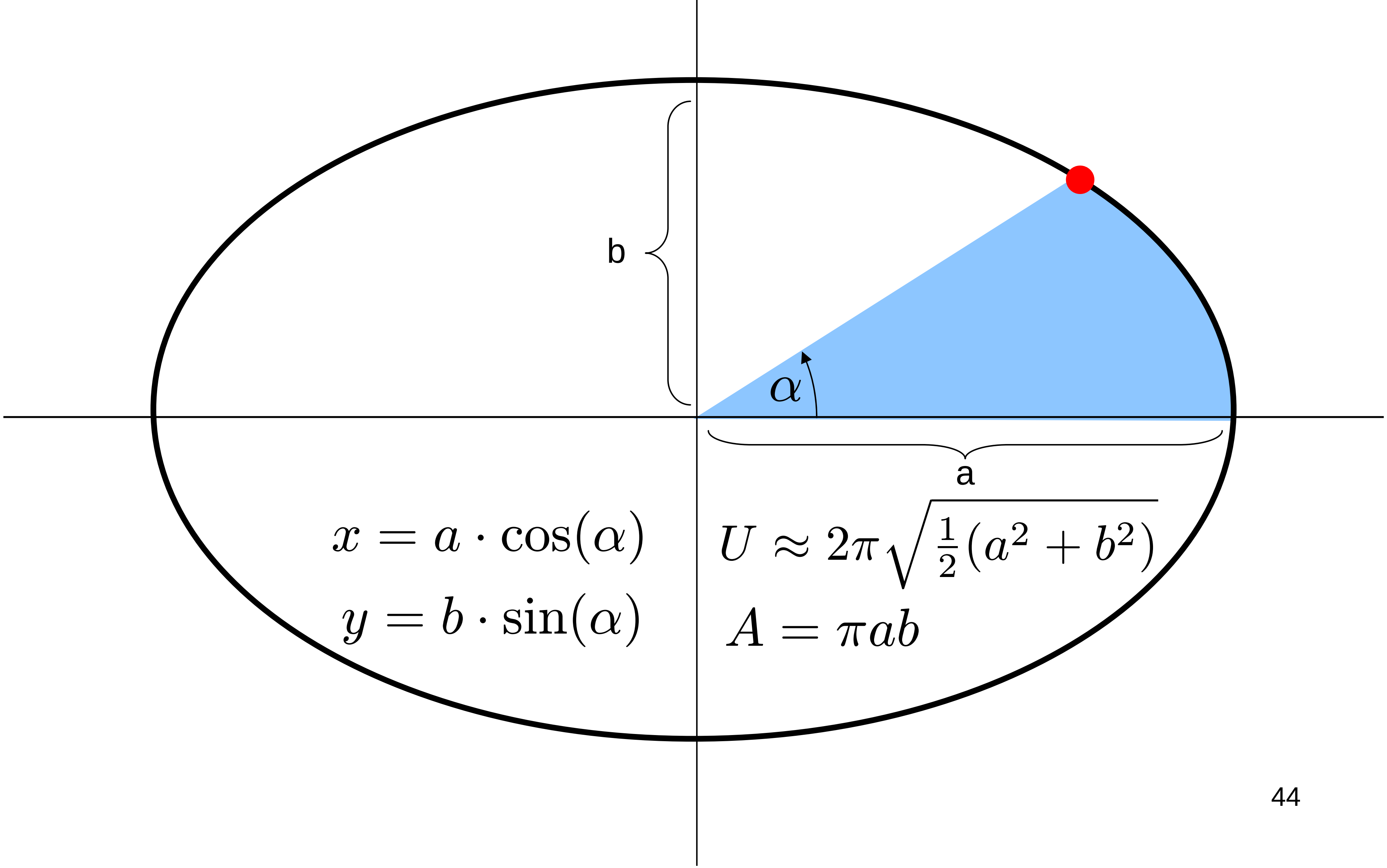

$$a = 5$$
$$b = 3$$

$$9x^2 + 25y^2 = 225$$

$$y = \sqrt{\frac{225-9x^2}{25}}$$

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

# Ellipse um (0,0) parametrisiert

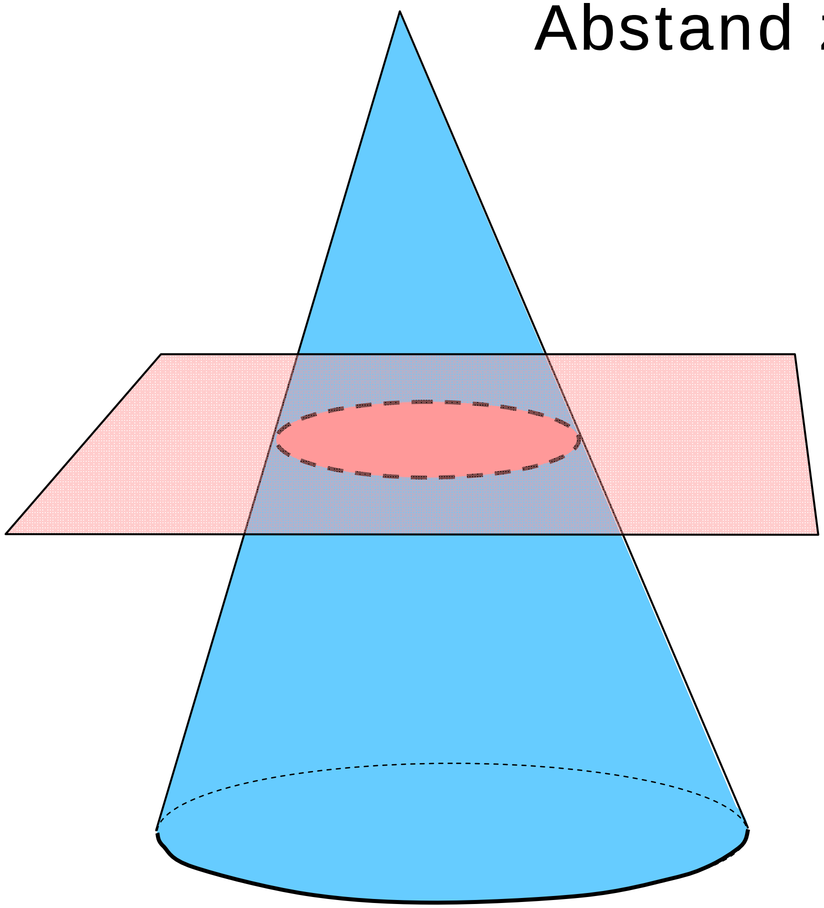



$$x = a \cdot \cos(\alpha)$$
$$y = b \cdot \sin(\alpha)$$

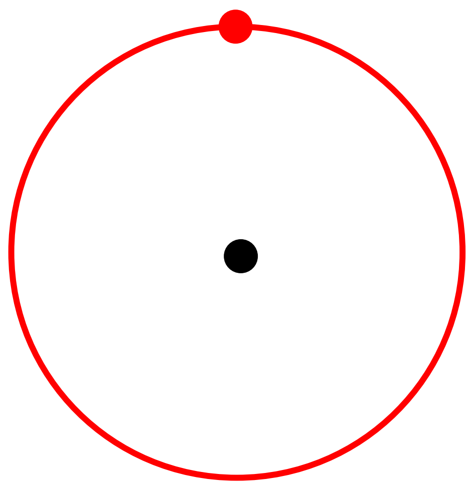
$$U \approx 2\pi\sqrt{\tfrac{1}{2}(a^2 + b^2)}$$
$$A = \pi a b$$

# Kegelschnitt: Kreis

Abstand zu einem Punkt ist konstant



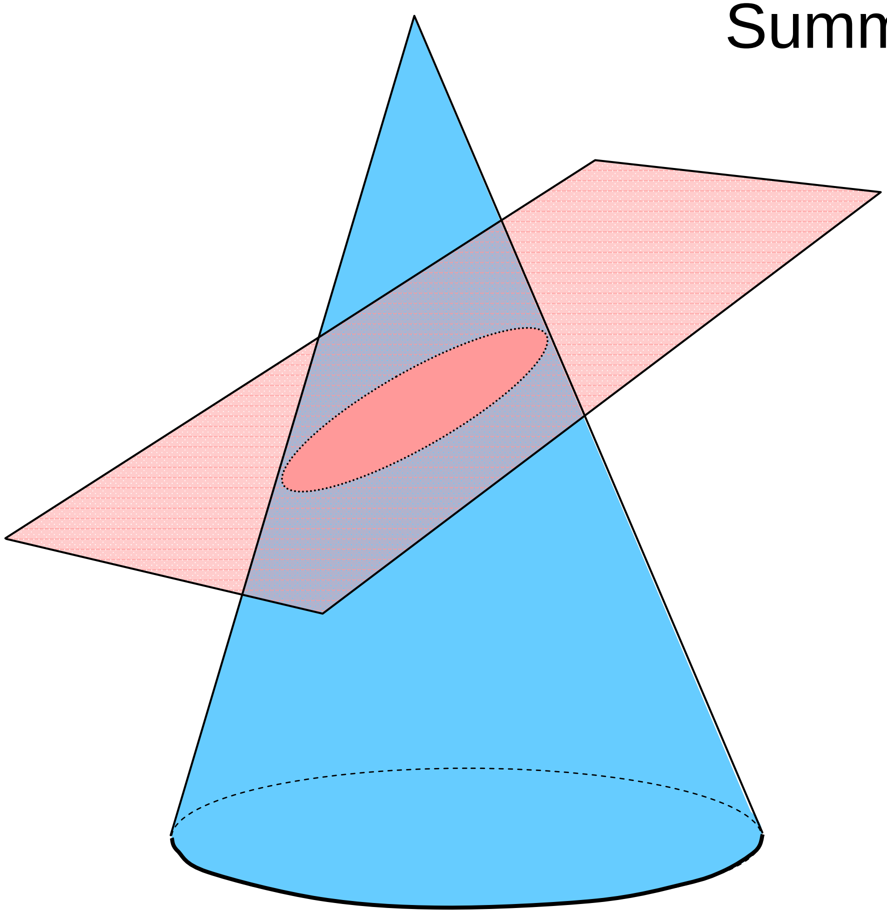
$$x^2 + y^2 = 1$$

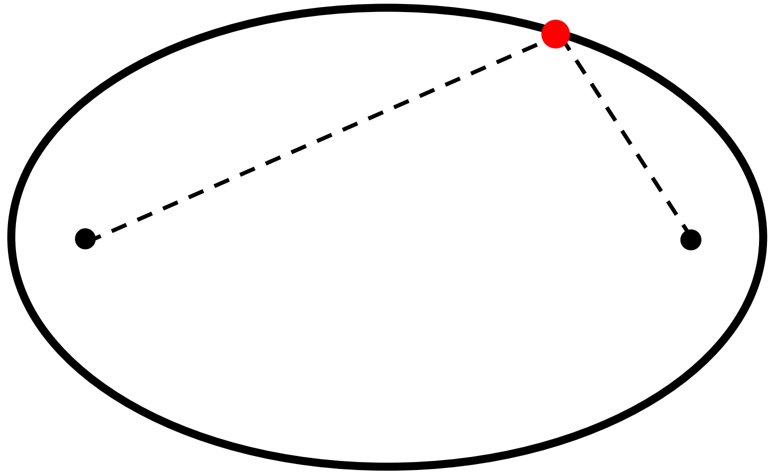


# Kegelschnitt: Ellipse

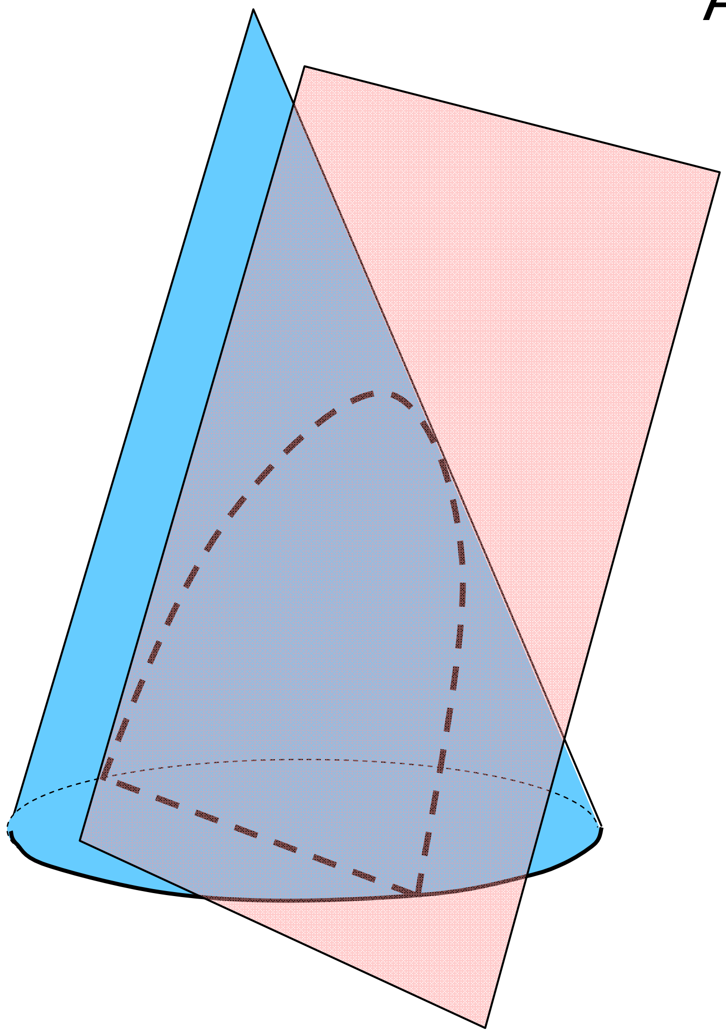
Summe der Abstände zu 2 Punkten
ist konstant

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$



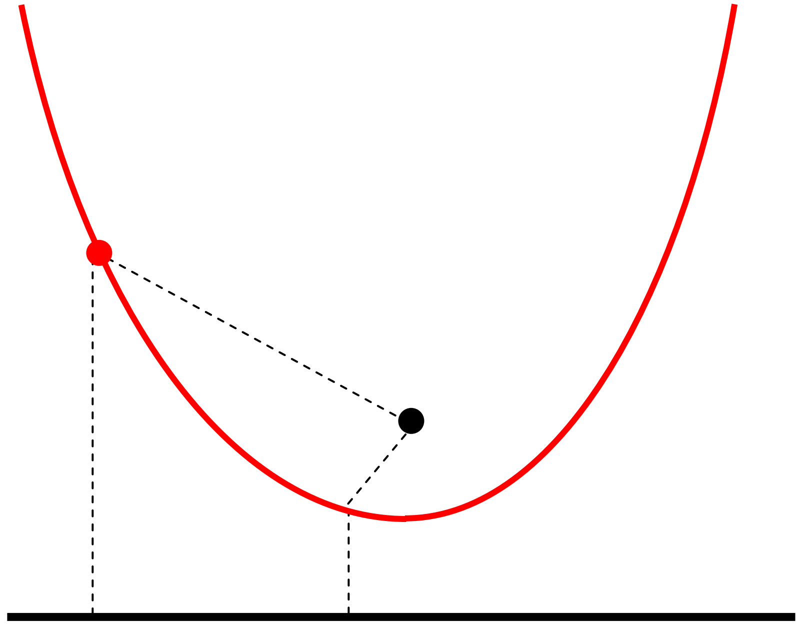


# Kegelschnitt: Parabel

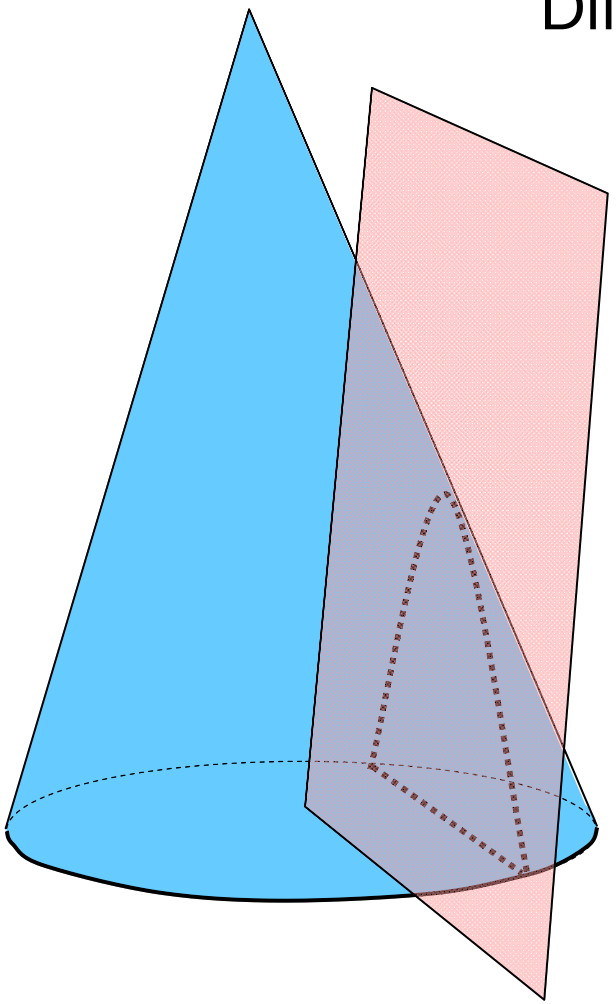


Abstand zu Punkt und Gerade
ist gleich

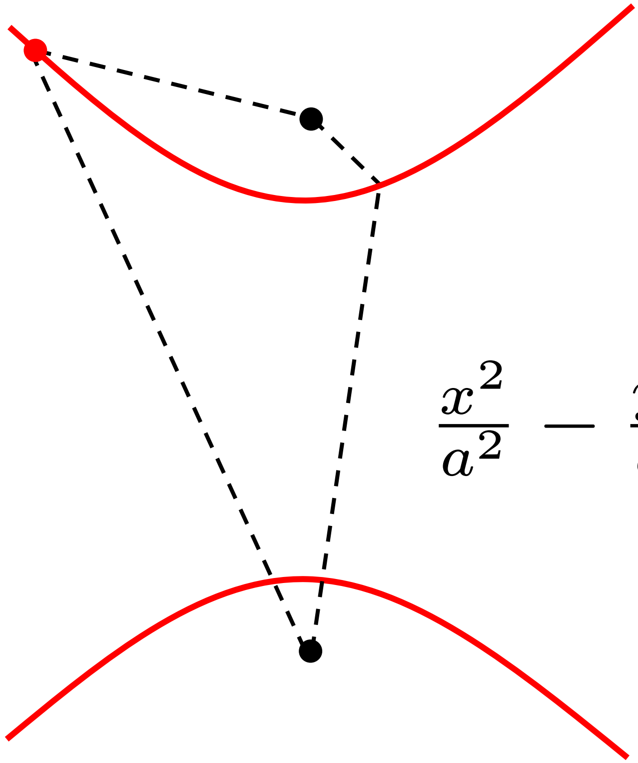
$$y = ax^2 + bx + c$$



# Kegelschnitt: Hyperbel

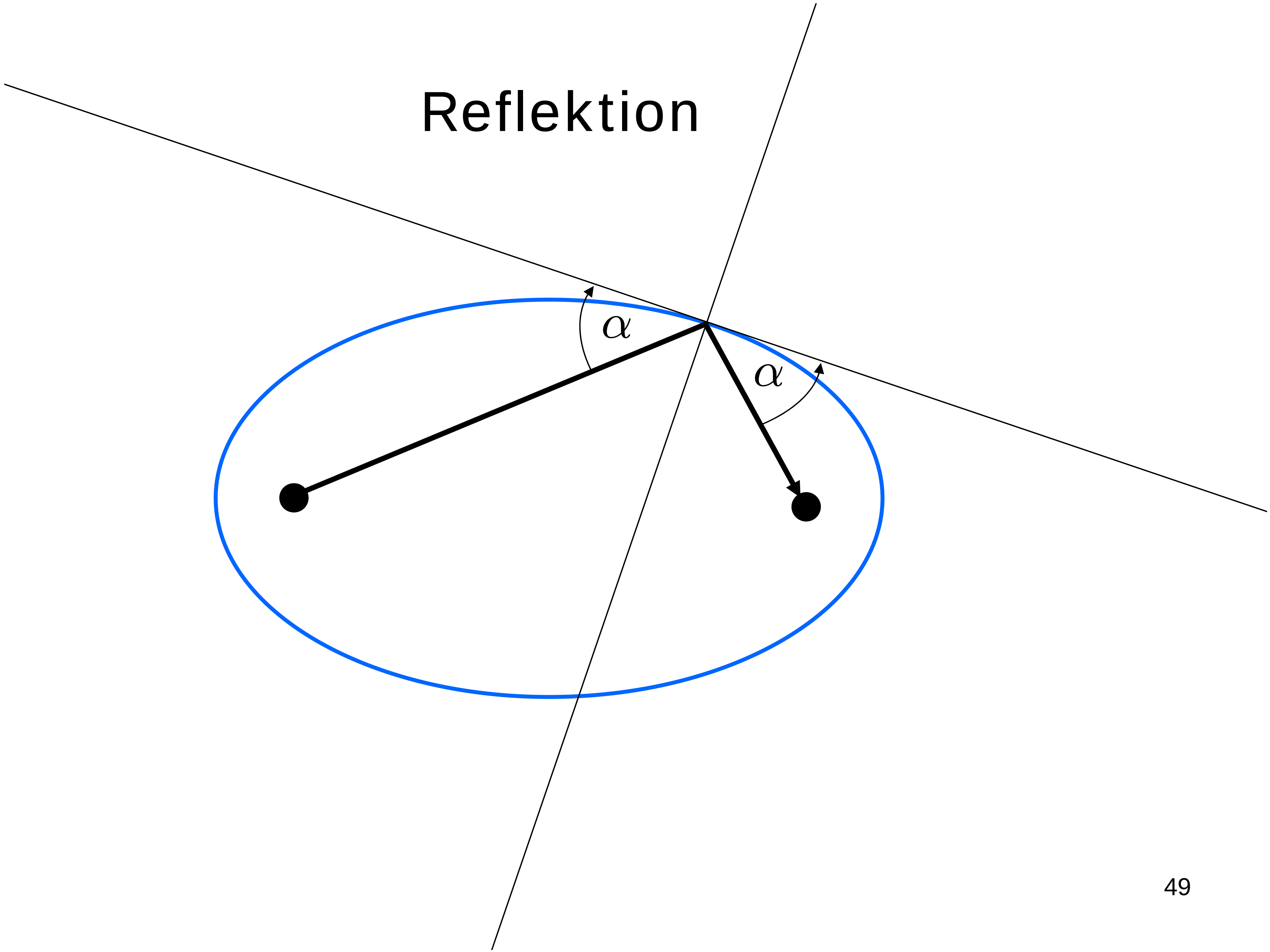


Differenz der Abstände zu 2 Punkten
ist konstant

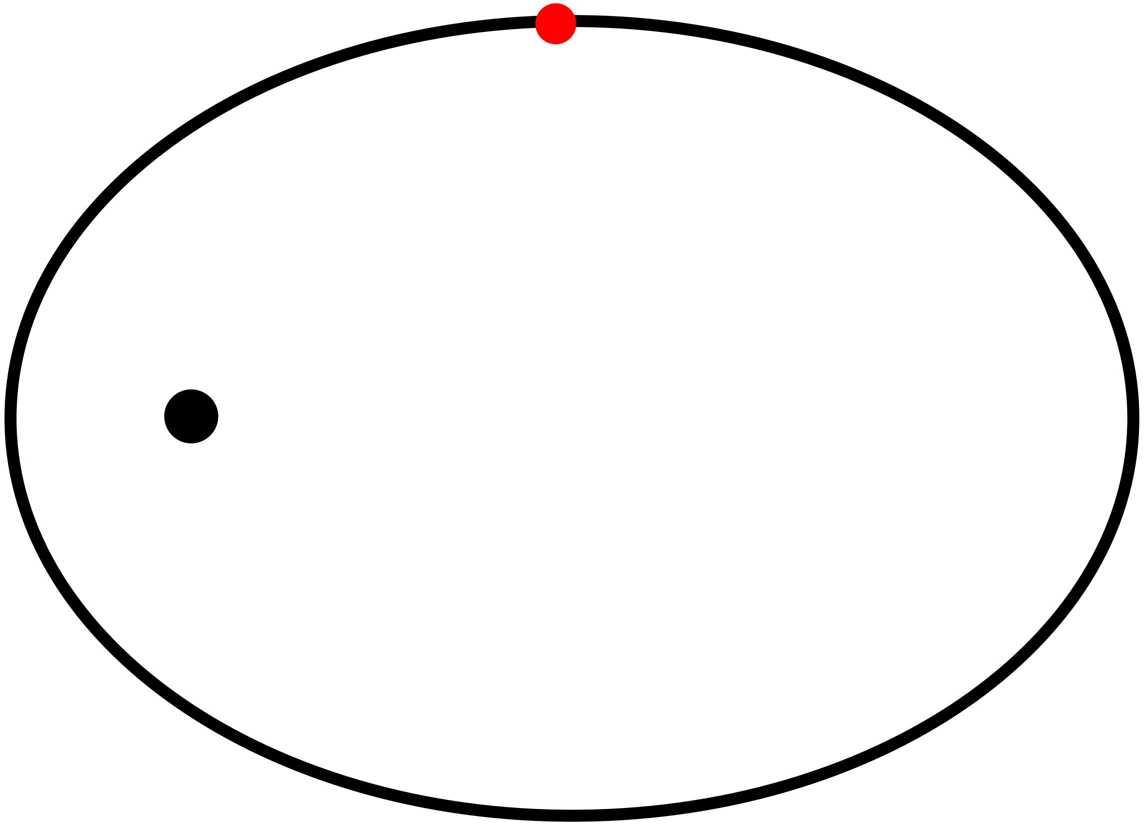


$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$$

# Reflektion

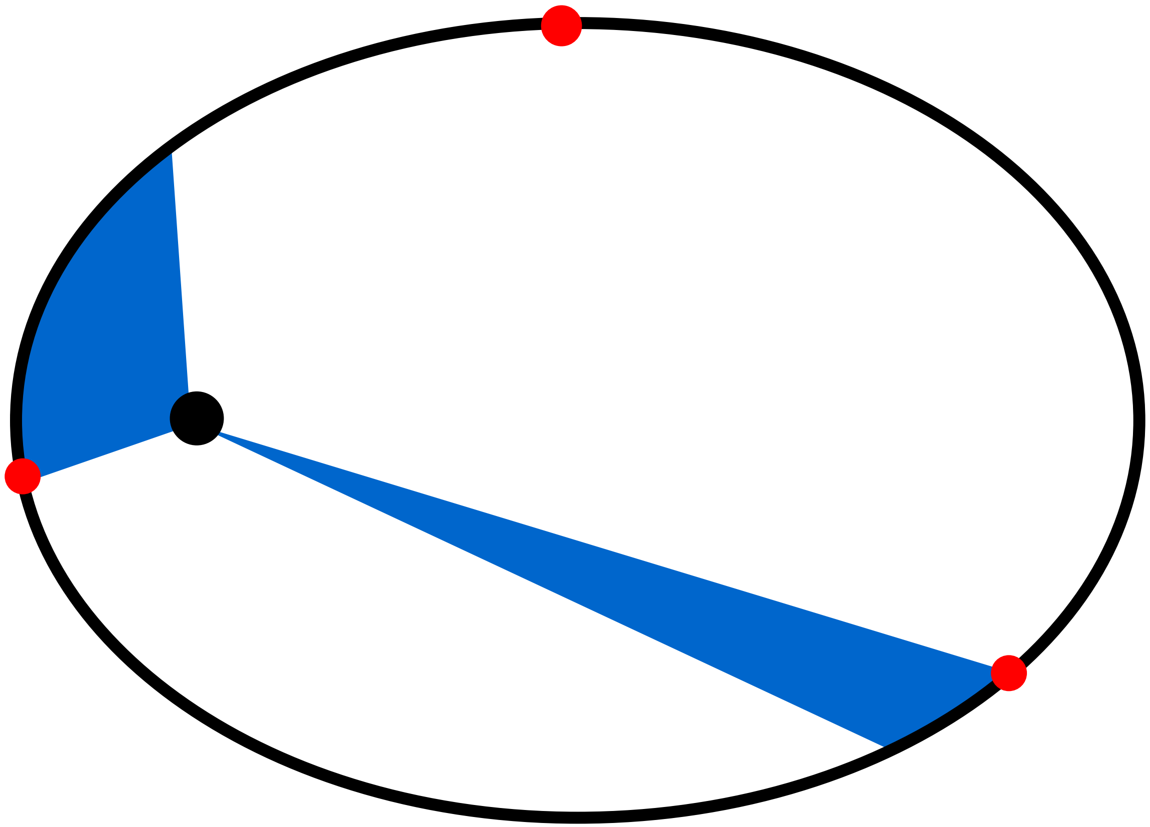


# 1. Keplersches Gesetz



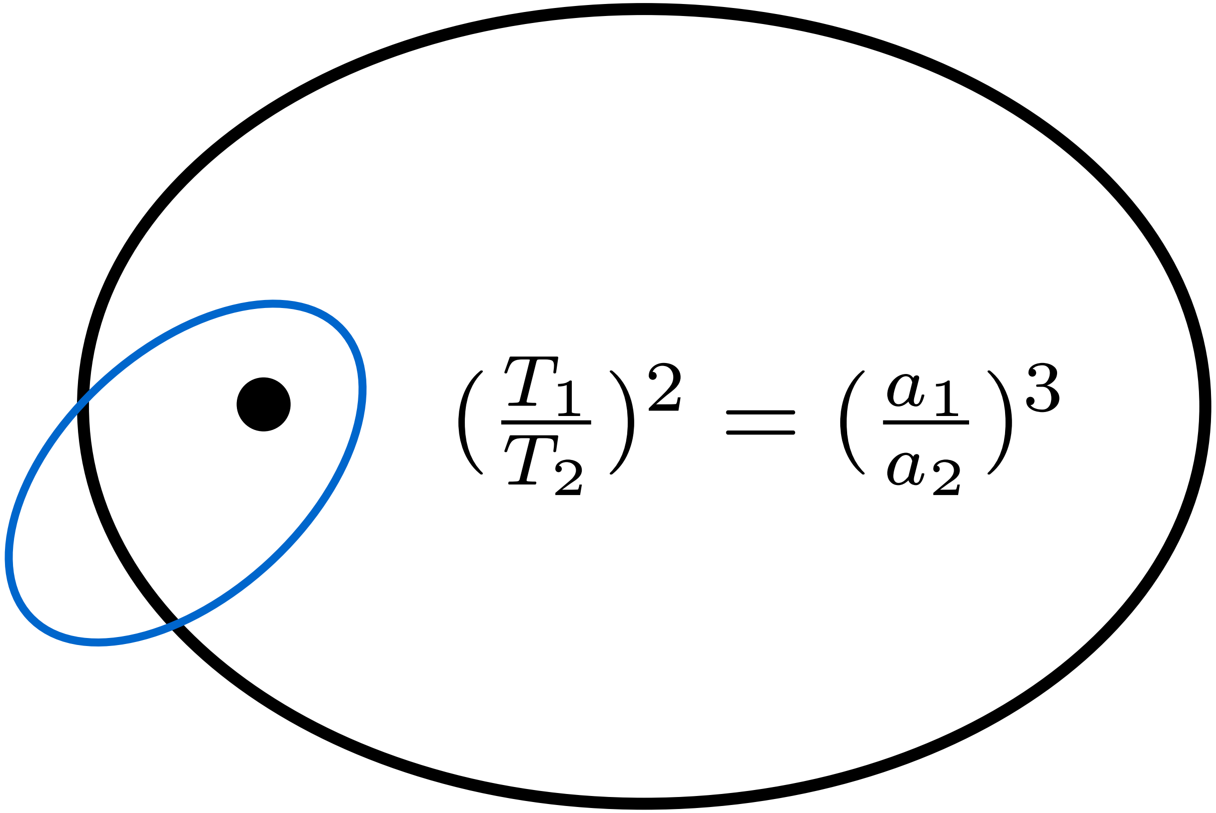
Die Planeten umkreisen die Sonne auf einer Ellipse

# 2. Keplersches Gesetz



In gleichen Zeiten überstreicht der Fahrstrahl gleiche Flächen

# 3. Keplersches Gesetz



$$\left(\frac{T_1}{T_2}\right)^2 = \left(\frac{a_1}{a_2}\right)^3$$

Die Quadrate der Umlaufzeiten verhalten sich
wie die Kuben der großen Halbachsen