Computergrafik SS 2010

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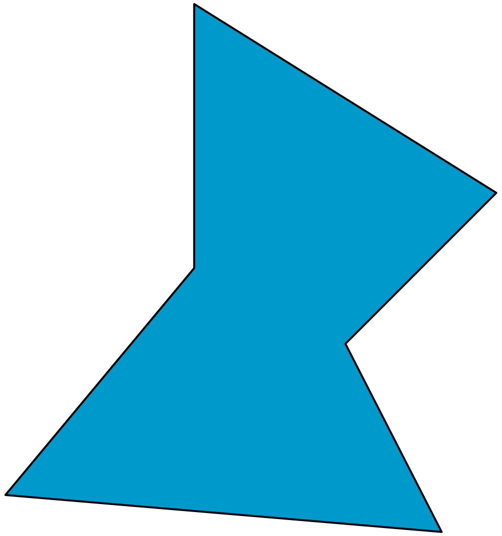
Kapitel 6:
2D-Transformationen

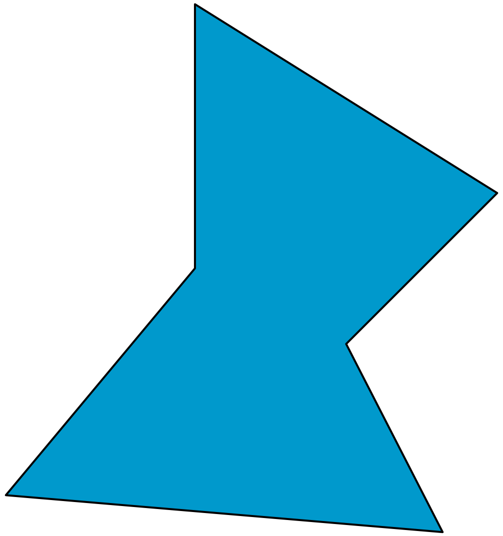
Translation



$$x := x + t_x$$

$$y := y + t_y$$

Skalierung



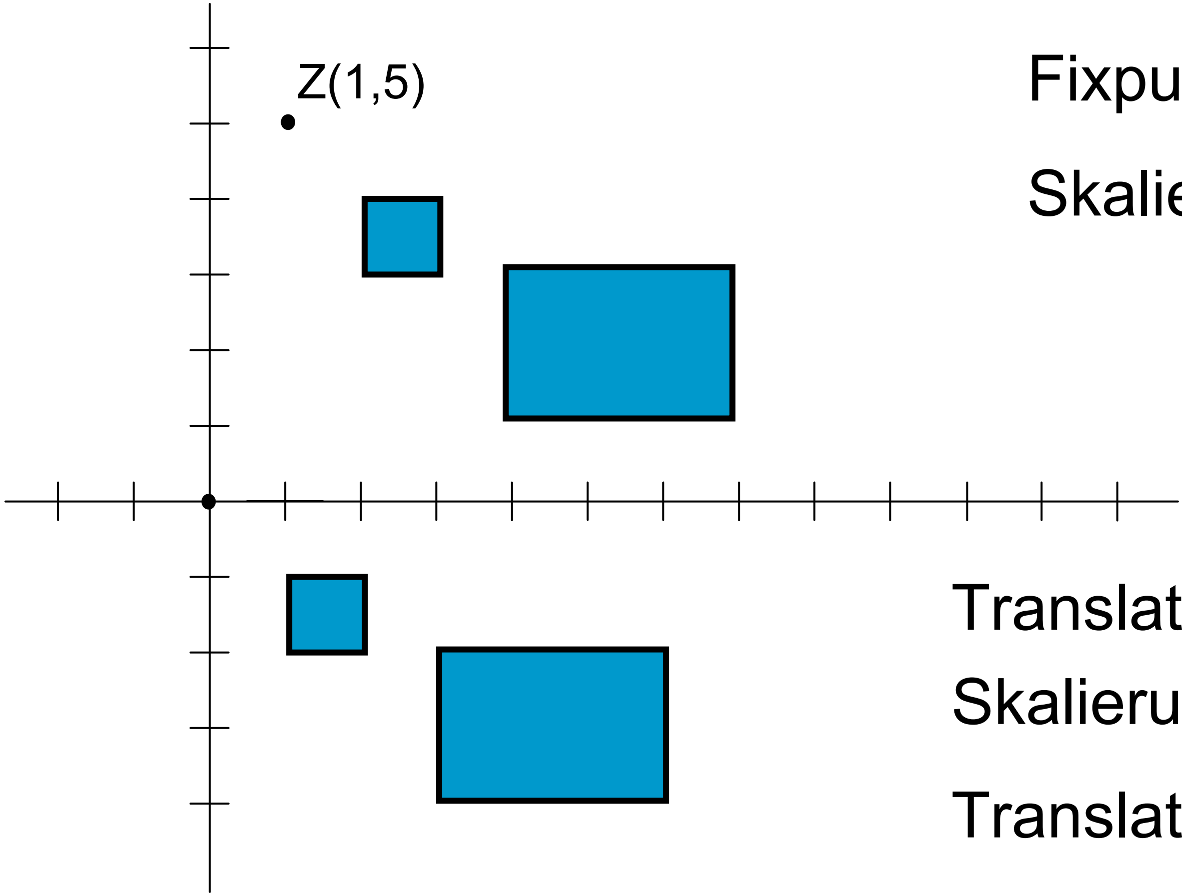
$$x := x \cdot s_x$$

$$y := y \cdot s_y$$

$s_x = s_y$ uniforme Skalierung

$s_x \neq s_y$ Verzerrung

Skalierung bzgl. Fixpunkt



Fixpunkt Z_x=1, Z_y=5

Skalierung s_x=3, s_y=2

Translation um $(-Z_x,-Z_y)$

Skalierung mit (s_x,s_y)

Translation um (Z_x,Z_y)

Skalierungsformel

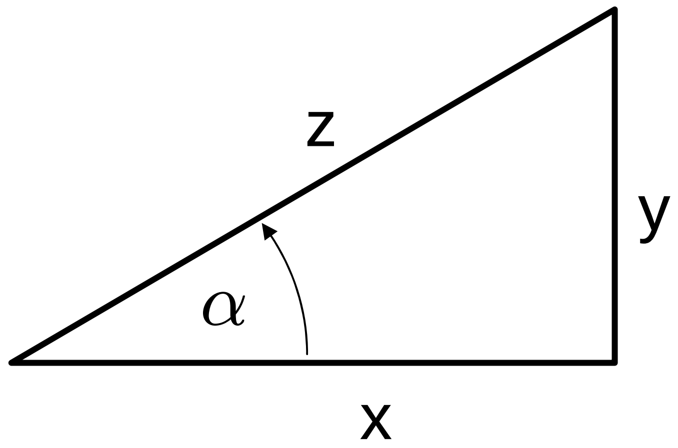
$$x' = (x - Z_x) \cdot s_x + Z_x$$

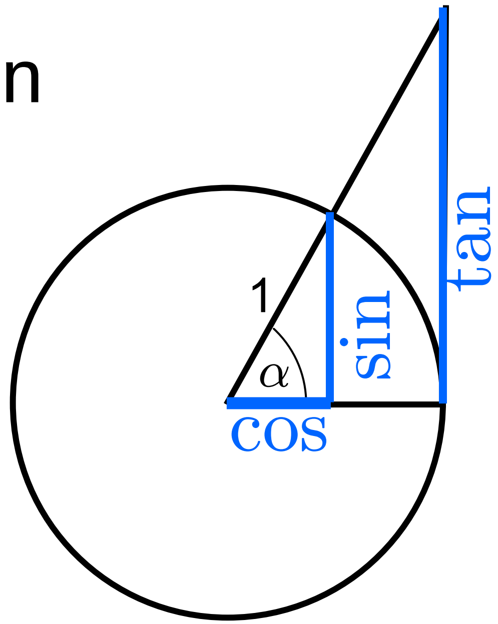
$$y' = (y - Z_y) \cdot s_y + Z_y$$

$$x' = x \cdot s_x - \underbrace{Z_x \cdot s_x + Z_x}_{d_x}$$

$$y' = y \cdot s_y - \underbrace{Z_y \cdot s_y + Z_y}_{d_y}$$

Trigonometrische Funktionen





$$\cos(\alpha) = \frac{x}{z} \qquad \sin(\alpha) = \frac{y}{z}$$

$$\tan(\alpha) = \frac{y}{x} \qquad \cot(\alpha) = \frac{x}{y}$$

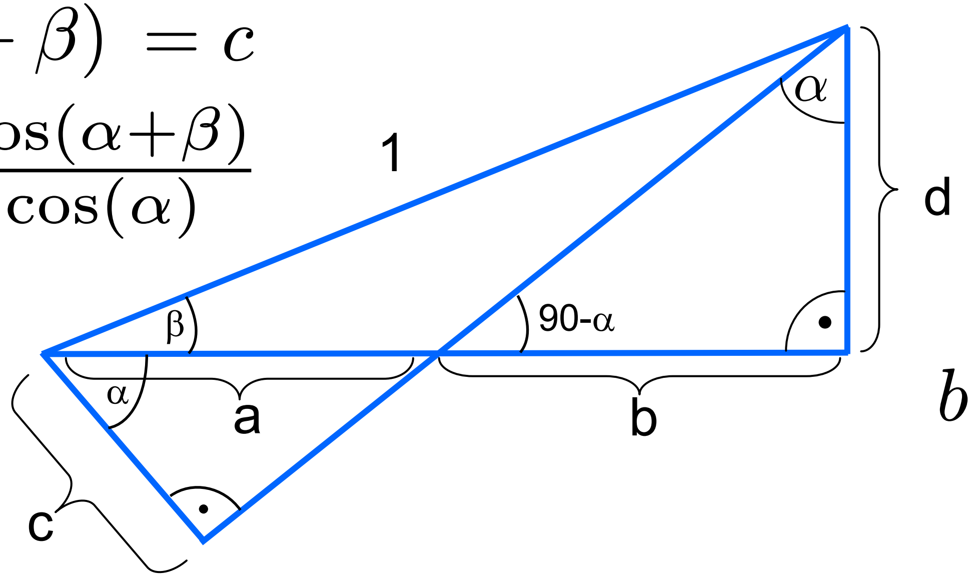
$$\tan(\alpha) = \frac{\sin(\alpha)}{\cos(\alpha)} \qquad \cot(\alpha) = \frac{1}{\tan(\alpha)}$$

Additionstheorem

$$\cos(\alpha) = c/a$$

$$\cos(\alpha+\beta) = c$$

$$a = \frac{\cos(\alpha+\beta)}{\cos(\alpha)}$$



$$\sin(\beta) = d$$

$$\tan(\alpha) = b/d$$

$$\tan(\alpha) = \frac{\sin(\alpha)}{\cos(\alpha)}$$

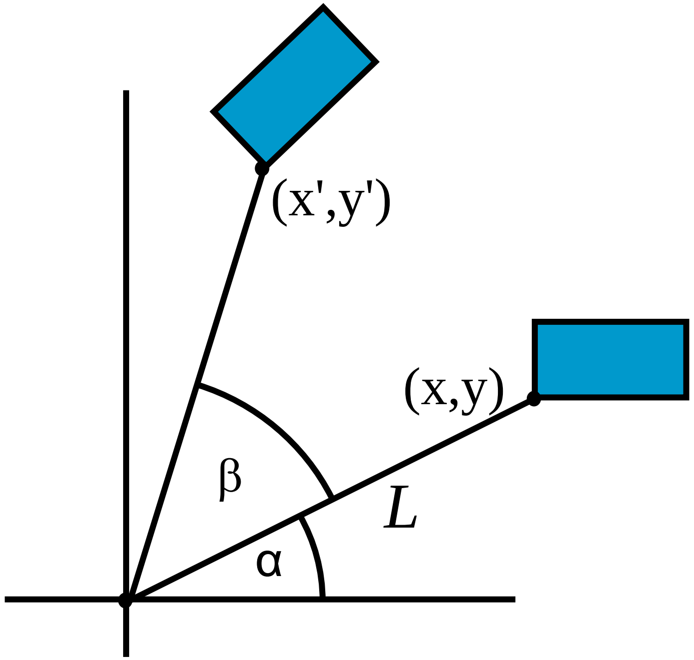
$$b = \frac{\sin(\alpha)}{\cos(\alpha)} \cdot \sin(\beta)$$

$$a + b = \cos(\beta) = \frac{cos(\alpha+\beta)}{cos(\alpha)} + sin(\beta) \cdot \frac{\sin(\alpha)}{cos(\alpha)}$$

$$cos(\alpha) \cdot \cos(\beta) = \cos(\alpha+\beta) + \sin(\alpha) \cdot \sin(\beta)$$

$$\cos(\alpha+\beta) = cos(\alpha) \cdot \cos(\beta) - \sin(\alpha) \cdot \sin(\beta)$$

Drehung

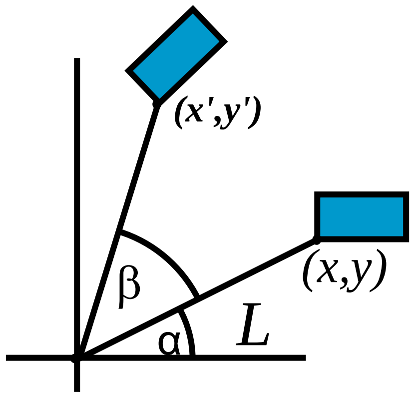


$$\cos(\alpha) = x/L$$

$$\sin(\alpha) = y/L$$

$$\cos(\alpha + \beta) = x'/L$$

$$\sin(\alpha + \beta) = y'/L$$



Formel für Drehung

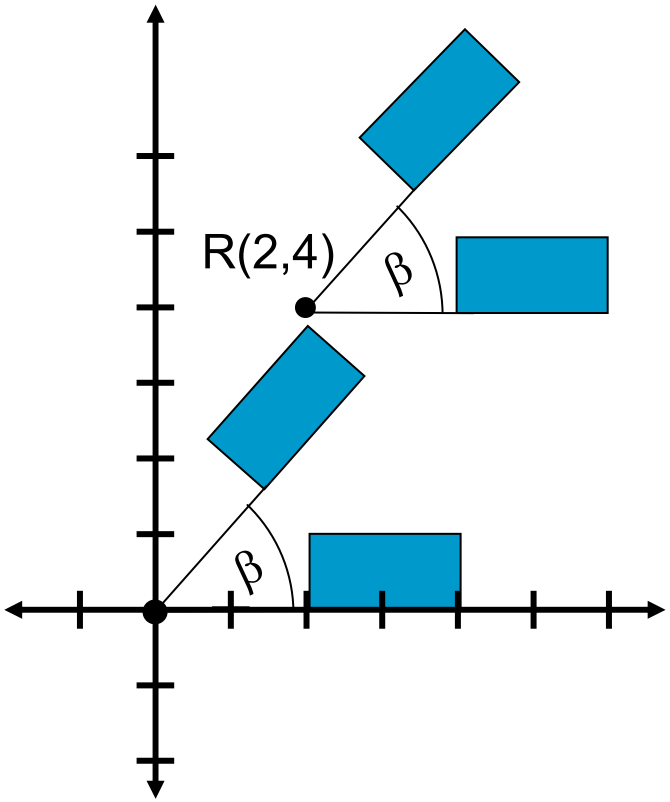
$$\cos(\alpha) = x/L \qquad \sin(\alpha) = y/L$$
$$\cos(\alpha+\beta) = x'/L \qquad \sin(\alpha+\beta) = y'/L$$

$$\sin(\alpha+\beta) = \cos(\beta)\cdot\sin(\alpha) + \sin(\beta)\cdot\cos(\alpha)$$
$$\cos(\alpha+\beta) = \cos(\beta)\cdot\cos(\alpha) - \sin(\beta)\cdot\sin(\alpha)$$

$$\cos(\alpha+\beta) = x'/L = \cos(\beta)\cdot\cos(\alpha) - \sin(\beta)\cdot\sin(\alpha)$$
$$x' = L\cdot\cos(\beta)\cdot x/L - \sin(\beta)\cdot y/L\cdot L$$
$$x' = x\cdot\cos(\beta) - y\cdot\sin(\beta)$$

$$\sin(\alpha+\beta) = y'/L = \cos(\beta)\cdot\sin(\alpha) + \sin(\beta)\cdot\cos(\alpha)$$
$$y' = L\cdot\cos(\beta)\cdot y/L + L\cdot\sin(\beta)x/L$$
$$y' = x\cdot\sin(\beta) + y\cdot\cos(\beta)$$

Rotation bzgl. Rotationszentrum



Translation um $(-R_x,-R_y)$

Rotation um β bzgl. (0,0)

Translation um (R_x,R_y)

Matrix für Rotation

$$x' := x \cdot \cos(\beta) - y \cdot \sin(\beta)$$

$$y' := x \cdot \sin(\beta) + y \cdot \cos(\beta)$$

$$\begin{pmatrix} x' \\ y' \end{pmatrix} := \begin{pmatrix} \cos(\beta) & -\sin(\beta) \\ \sin(\beta) & \cos(\beta) \end{pmatrix} \cdot \begin{pmatrix} x \\ y \end{pmatrix}$$

$$(x' \; y') := (x \; y) \cdot \begin{pmatrix} \cos(\beta) & \sin(\beta) \\ -\sin(\beta) & \cos(\beta) \end{pmatrix}$$

$$A \cdot B = (B^T \cdot A^T)^T$$

Matrix für Skalierung

$$x' := x \cdot s_x$$

$$y' := y \cdot s_y$$

$$\begin{pmatrix} x' \\ y' \end{pmatrix} := \begin{pmatrix} s_x & 0 \\ 0 & s_y \end{pmatrix} \cdot \begin{pmatrix} x \\ y \end{pmatrix}$$

Matrix für Translation

$$x' := x + t_x$$

$$y' := y + t_y$$

$$\begin{pmatrix} x' \\ y' \end{pmatrix} := \begin{pmatrix} ? & ? \\ ? & ? \end{pmatrix} \cdot \begin{pmatrix} x \\ y \end{pmatrix}$$

Homogene Koordinaten

$P = \begin{pmatrix} x \\ y \end{pmatrix}$ hat homogene Koordinaten $\begin{pmatrix} x \cdot w \\ y \cdot w \\ w \end{pmatrix}$

mit w ≠0

Zu den homogenen Koordinaten $\begin{pmatrix} x \\ y \\ w \end{pmatrix}$ gehört $P = \begin{pmatrix} x/w \\ y/w \end{pmatrix}$

Richtungsvektor $\vec{v} = \begin{pmatrix} x \\ y \end{pmatrix}$ hat homogene Koordinaten $\begin{pmatrix} x \\ y \\ 0 \end{pmatrix}$

Zum Punkt $\begin{pmatrix} 3 \\ 4 \end{pmatrix} \epsilon \mathbb{R}^2$ gehört die Ursprungsgerade $\begin{pmatrix} 3 \cdot w \\ 4 \cdot w \\ w \end{pmatrix} \epsilon \mathbb{R}^3$

Matrix für Translation

$$\begin{pmatrix} x' \\ y' \\ w' \end{pmatrix} := \begin{pmatrix} 1 & 0 & t_x \\ 0 & 1 & t_y \\ 0 & 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} x \\ y \\ 1 \end{pmatrix}$$

$$:= \begin{pmatrix} x + t_x \\ y + t_y \\ 1 \end{pmatrix}$$

Beispiel für Translation

$$\begin{pmatrix} 7 \\ 8 \\ 1 \end{pmatrix} := \begin{pmatrix} 1 & 0 & 2 \\ 0 & 1 & 2 \\ 0 & 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} 5 \\ 6 \\ 1 \end{pmatrix}$$

$$\begin{pmatrix} 21 \\ 24 \\ 3 \end{pmatrix} := \begin{pmatrix} 1 & 0 & 2 \\ 0 & 1 & 2 \\ 0 & 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} 15 \\ 18 \\ 3 \end{pmatrix}$$

Matrix für Skalierung

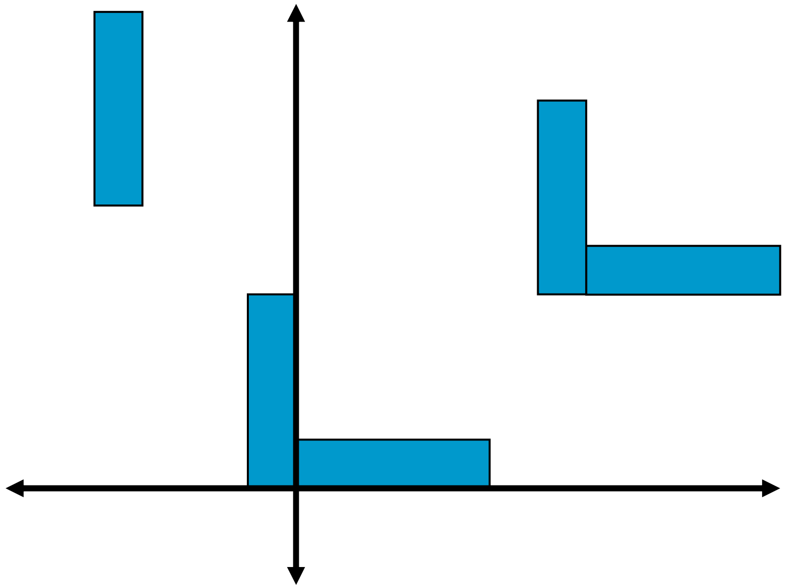
$$\begin{pmatrix} x' \\ y' \\ w' \end{pmatrix} := \begin{pmatrix} s_x & 0 & 0 \\ 0 & s_y & 0 \\ 0 & 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} x \\ y \\ 1 \end{pmatrix}$$

Matrix für Rotation

$$\begin{pmatrix} x' \\ y' \\ w' \end{pmatrix} := \begin{pmatrix} \cos(\beta) & -\sin(\beta) & 0 \\ \sin(\beta) & \cos(\beta) & 0 \\ 0 & 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} x \\ y \\ 1 \end{pmatrix}$$

Verknüpfung von Transformationen

- assoziativ: A•B•C = (A•B)•C = A•(B •C)
- nicht kommunativ: A • B ≠ B • A
- Drehung um 90° + Verschieben um (4,3) ≠ Verschieben um (4,3) + Drehung um 90°



Rotation bzgl (3,5) um 60°

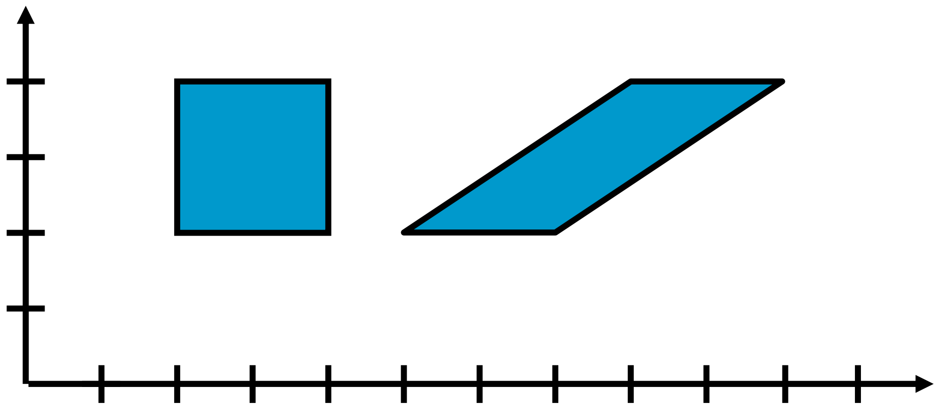
$$A = \begin{pmatrix} 1 & 0 & -3 \\ 0 & 1 & -5 \\ 0 & 0 & 1 \end{pmatrix} \qquad C = \begin{pmatrix} 1 & 0 & 3 \\ 0 & 1 & 5 \\ 0 & 0 & 1 \end{pmatrix}$$

$$B = \begin{pmatrix} 0.5000000 & -0.8660254 & 0.000000 \\ 0.8660254 & 0.5000000 & 0.000000 \\ 0.0000000 & 0.0000000 & 1.000000 \end{pmatrix}$$

$$D = C \cdot B \cdot A \; =$$

$$\begin{pmatrix} 0.5000000 & -0.8660254 & 2.8301270 \\ 0.8660254 & 0.5000000 & -0.0980762 \\ 0.0000000 & 0.0000000 & 1.0000000 \end{pmatrix}$$

Matrix für Scherung in x-Richtung

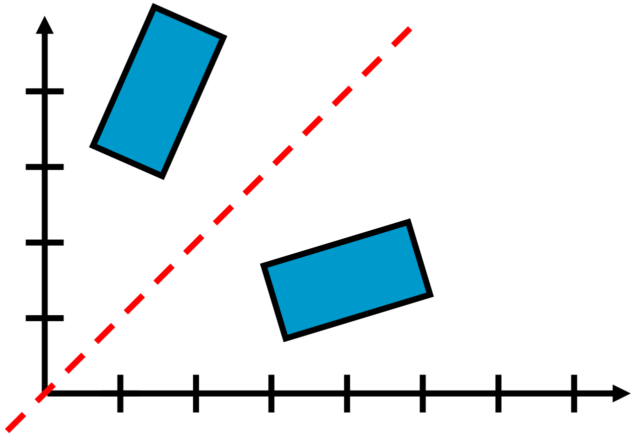


$$x' := x + m \cdot y$$

$$y' := y$$

$$\begin{pmatrix} x' \\ y' \\ w' \end{pmatrix} := \begin{pmatrix} 1 & m & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} x \\ y \\ 1 \end{pmatrix}$$

Matrix für Spiegelung an Hauptdiagonale

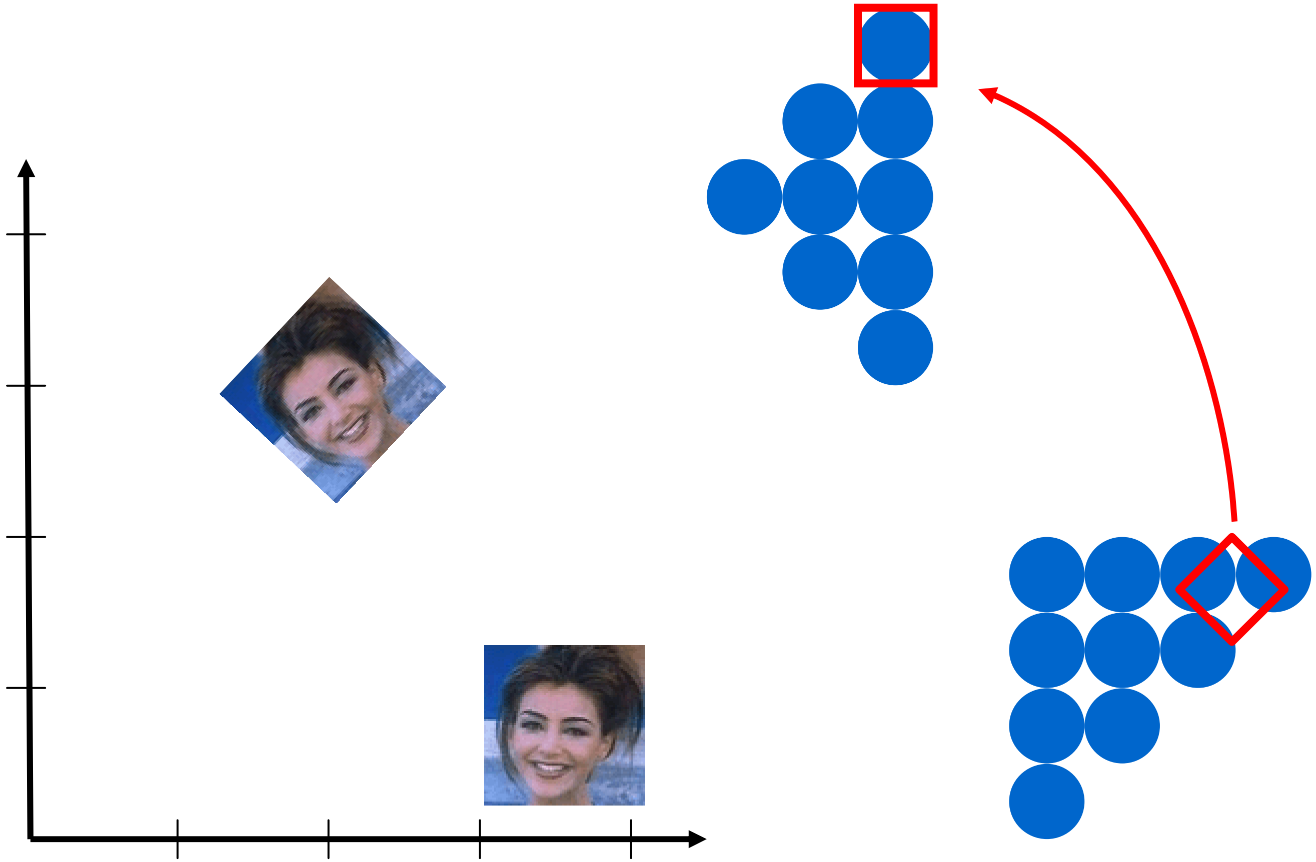


$$x' := y$$

$$y' := x$$

$$\begin{pmatrix} x' \\ y' \\ w' \end{pmatrix} := \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} x \\ y \\ 1 \end{pmatrix}$$

Pixel-Transformation



Transformations-Implementation

Java-Applet:

~cg/2010/skript/Applets/2D-trafo/App.html