

Computergrafik SS 2010

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Kapitel 13:  
3D-Transformationen

# Einsatzgebiet

- Platzierung von Objekten in der Szene
- Berechnung der Projektion

# Translation

$$(x', y', z') := (x + t_x, y + t_y, z + t_z)$$

$$T(t_x, t_y, t_z) = \begin{pmatrix} 1 & 0 & 0 & t_x \\ 0 & 1 & 0 & t_y \\ 0 & 0 & 1 & t_z \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

# Skalierung

Fixpunkt im Ursprung:

$$(x', y', z') := (x \cdot s_x, y \cdot s_y, z \cdot s_z)$$

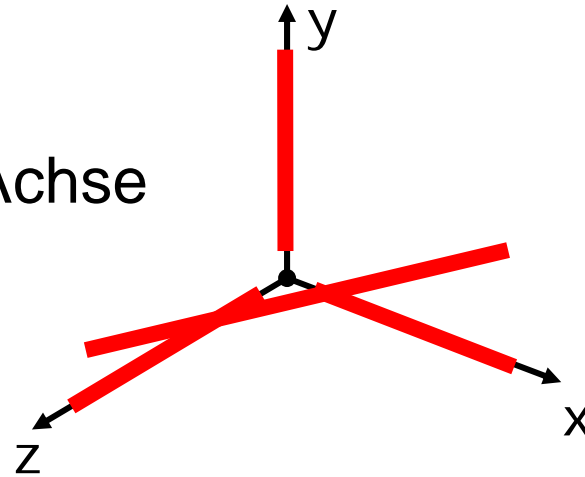
$$S(s_x, s_y, s_z) = \begin{pmatrix} s_x & 0 & 0 & 0 \\ 0 & s_y & 0 & 0 \\ 0 & 0 & s_z & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

Fixpunkt bei  $Z_x, Z_y, Z_z$ :

$$T(Z_x, Z_y, Z_z) \cdot S(s_x, s_y, s_z) \cdot T(-Z_x, -Z_y, -Z_z)$$

# Rotation

- um z-Achse
- um x-Achse
- um y-Achse
- um beliebige Achse

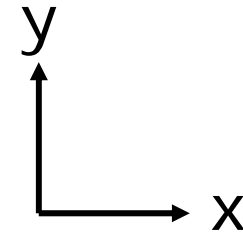


## Rotation um z-Achse

$$x' := x \cdot \cos(\delta) - y \cdot \sin(\delta)$$

$$y' := x \cdot \sin(\delta) + y \cdot \cos(\delta)$$

$$z' := z$$



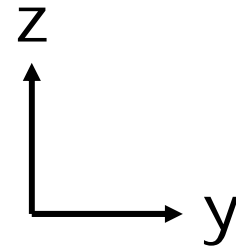
$$R_z(\delta) = \begin{pmatrix} \cos(\delta) & -\sin(\delta) & 0 & 0 \\ \sin(\delta) & \cos(\delta) & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

## Rotation um x-Achse

$$x' := x$$

$$y' := y \cdot \cos(\delta) - z \cdot \sin(\delta)$$

$$z' := y \cdot \sin(\delta) + z \cdot \cos(\delta)$$



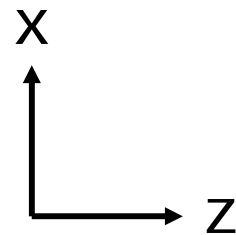
$$R_x(\delta) = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos(\delta) & -\sin(\delta) & 0 \\ 0 & \sin(\delta) & \cos(\delta) & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

## Rotation um y-Achse

$$x' := z \cdot \sin(\delta) + x \cdot \cos(\delta)$$

$$y' := y$$

$$z' := z \cdot \cos(\delta) - x \cdot \sin(\delta)$$



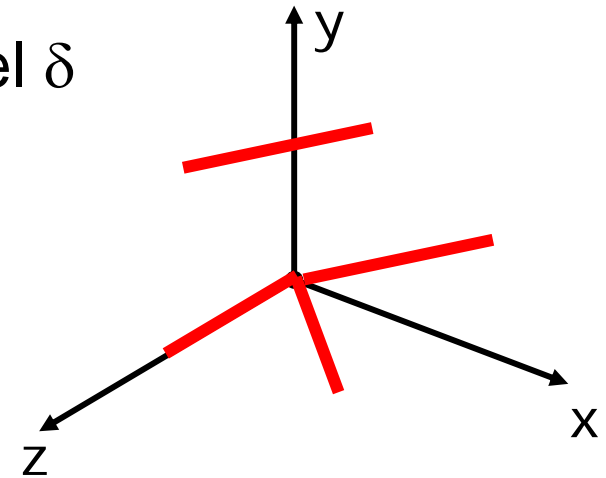
$$R_y(\delta) = \begin{pmatrix} \cos(\delta) & 0 & \sin(\delta) & 0 \\ 0 & 1 & 0 & 0 \\ -\sin(\delta) & 0 & \cos(\delta) & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$



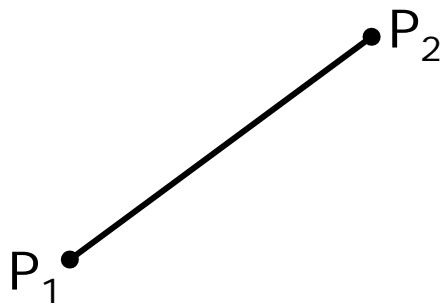
# Rotation um beliebige Achse

Punkt  $P$     Drehwinkel  $\delta$     Drehachse  $P_2-P_1$

1. Translation in den Ursprung
2. Rotation um die  $x$ -Achse in die  $xz$ -Ebene
3. Rotation um die  $y$ -Achse in die  $z$ -Achse
4. Rotation um die  $z$ -Achse mit Winkel  $\delta$
5. Inversion von Schritt 3
6. Inversion von Schritt 2
7. Inversion von Schritt 1



# Drehachse



A diagram showing two points,  $P_1$  and  $P_2$ , connected by a line segment.  $P_1$  is at the bottom left, and  $P_2$  is at the top right. A vector  $\vec{v}$  is represented by this line segment, pointing from  $P_1$  to  $P_2$ .

$$\vec{v} = P_2 - P_1 = \begin{pmatrix} x_2 - x_1 \\ y_2 - y_1 \\ z_2 - z_1 \end{pmatrix}$$

$$|\vec{v}| = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}$$

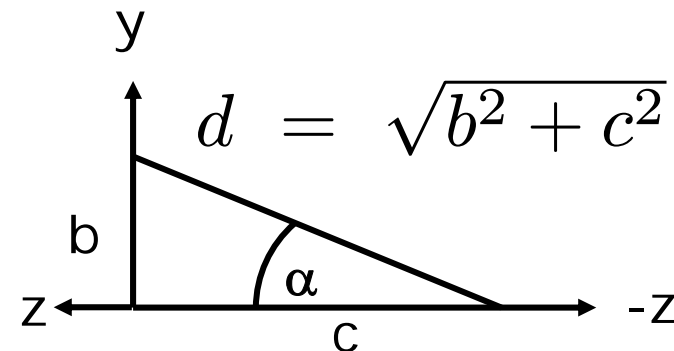
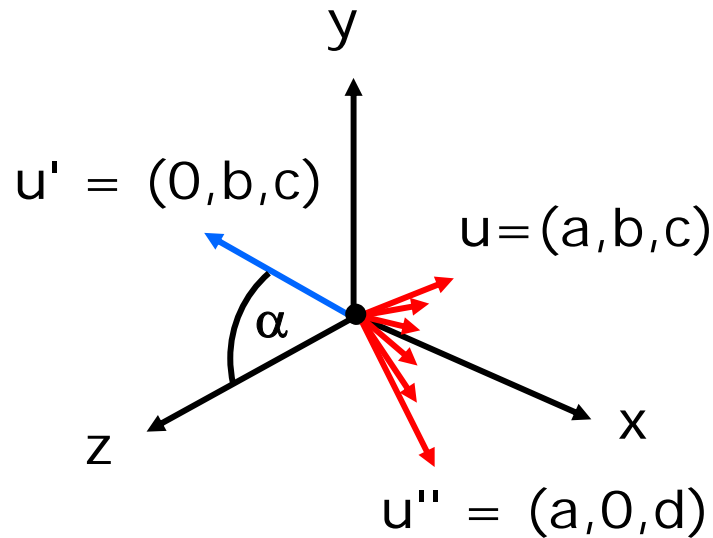
$$\vec{u} = \frac{\vec{v}}{|\vec{v}|} = \begin{pmatrix} a \\ b \\ c \end{pmatrix} \quad |\vec{u}| = 1$$

$$a = \frac{x_2 - x_1}{|\vec{v}|} \quad b = \frac{y_2 - y_1}{|\vec{v}|} \quad c = \frac{z_2 - z_1}{|\vec{v}|}$$

# 1.) Translation in den Ursprung

$$T(-x_1, -y_1, -z_1)$$

## 2.) Rotation um x-Achse in xz-Ebene

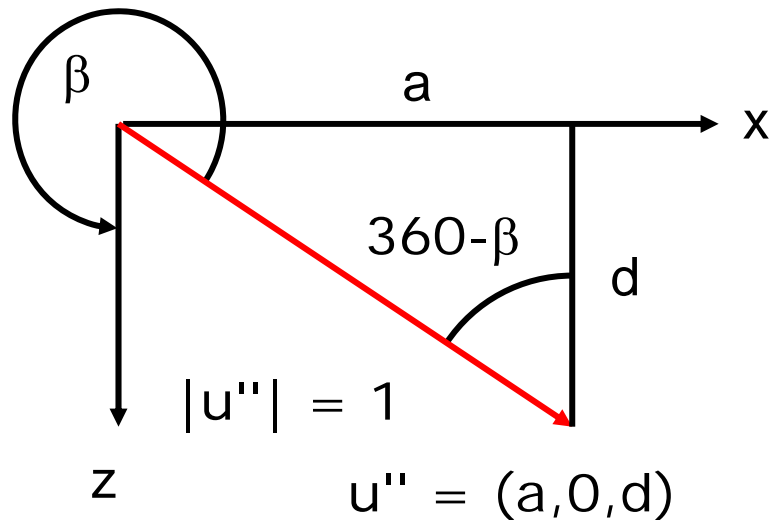


$$\cos(\alpha) = c/d$$

$$\sin(\alpha) = b/d$$

$$R_x(\alpha) = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & c/d & -b/d & 0 \\ 0 & b/d & c/d & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

### 3.) Rotation um y-Achse in z-Achse



$$R_y(\beta) = \begin{pmatrix} d & 0 & -a & 0 \\ 0 & 1 & 0 & 0 \\ a & 0 & d & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$$\Rightarrow \cos(\beta) = \cos(360^\circ - \beta) = d$$

$$\Rightarrow \sin(\beta) = -\sin(360^\circ - \beta) = -a$$

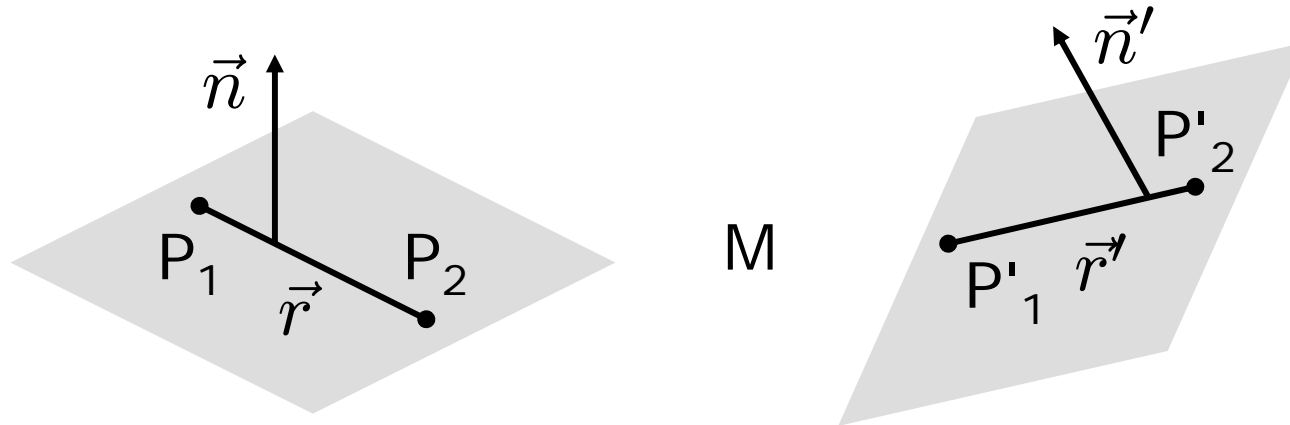
## 4.) Rotation um die z-Achse

$$R_z(\delta) = \begin{pmatrix} \cos(\delta) & -\sin(\delta) & 0 & 0 \\ \sin(\delta) & \cos(\delta) & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

# 1. - 7.) Gesamttransformation

$$R(\vec{v}, \delta) = T(-P_1) \cdot R_x(\alpha) \cdot R_y(\beta) \cdot R_z(\delta) \cdot R_y^{-1}(\beta) \cdot R_x^{-1}(\alpha) \cdot T(P_1).$$

# Transformation einer Normalen



$$\begin{aligned} \vec{r} &= P_2 - P_1 & P'_1 &= M \cdot P_1 & P'_2 &= M \cdot P_2 \\ \vec{r}' &= P'_2 - P'_1 \\ &= M \cdot P_2 - M \cdot P_1 = M \cdot (P_2 - P_1) = M \cdot \vec{r} \end{aligned}$$

$$\vec{n}^T \cdot \vec{r} = 0 \quad M \cdot \vec{r} = \vec{r}' \quad \vec{n}'^T \cdot \vec{r}' \stackrel{!}{=} 0$$



# Umformungen

$$\vec{n}^T \cdot r = 0$$

$$\vec{n}^T \cdot M^{-1} \cdot M \cdot r = 0$$

$$((M^{-1})^T \cdot \vec{n})^T \cdot \underbrace{M \cdot r}_{\vec{r}'} = 0$$

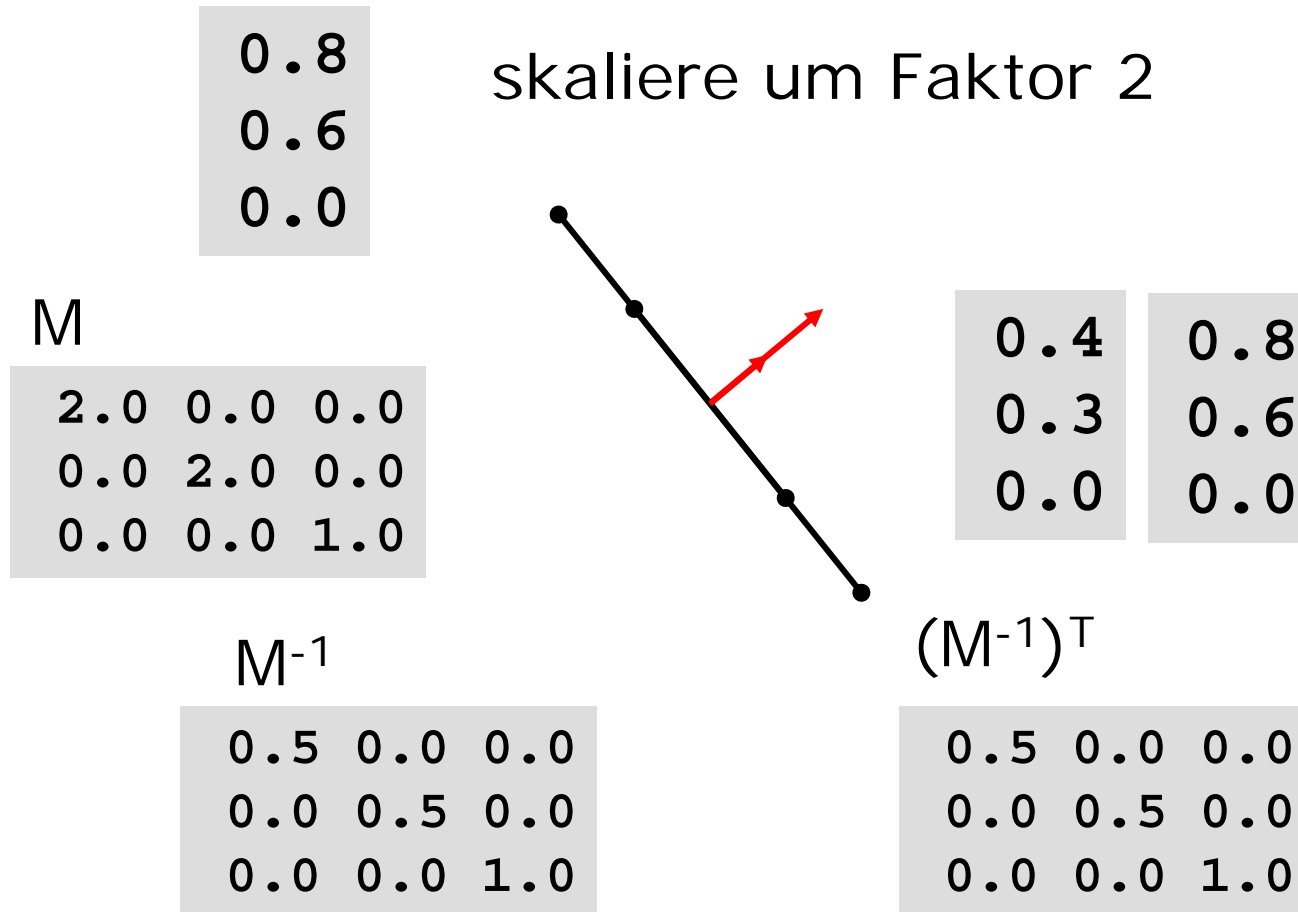
$$\vec{n}'^T \cdot \vec{r}' = 0$$

$$((M^{-1})^T \cdot \vec{n})^T = \vec{n}'^T$$

$$((M^{-1})^T \cdot \vec{n}) = \vec{n}'$$

⇒ transformiere den Normalenvektor  
mit der transponierten Inversen !

# Skalierung einer Normalen



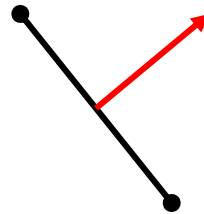
# Rotation einer Normalen

0.8  
0.6  
0.0

drehe um  $25^\circ$

M

0.91 -0.42 0.0  
0.42 0.91 0.0  
0.0 0.0 1.0



0.48  
0.88  
0.00

$M^{-1}$

0.91 0.42 0.0  
-0.42 0.91 0.0  
0.0 0.0 1.0

$(M^{-1})^T$

0.91 -0.42 0.0  
0.42 0.91 0.0  
0.0 0.0 1.0

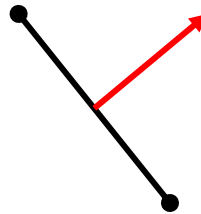
# Translation einer Normalen

0.8  
0.6  
0.0

verschiebe um (5,2)



0.8  
0.6  
0.0



$M$

$M^{-1}$

$(M^{-1})^T$

1 0 5  
0 1 2  
0 0 1

1 0 -5  
0 1 -2  
0 0 1

1 0 0  
0 1 0  
-5 -2 1