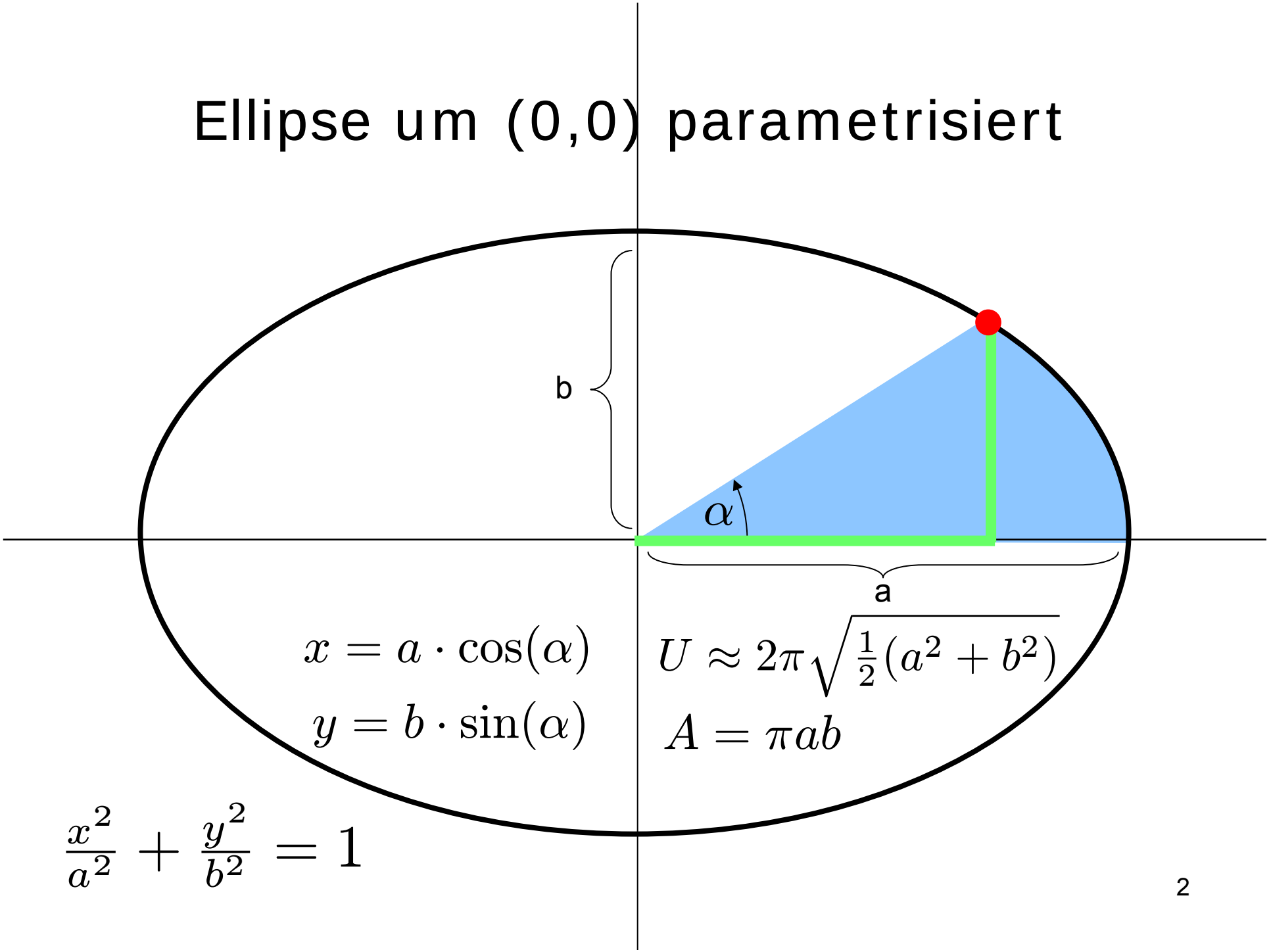
Computergrafik SS 2014

Oliver Vornberger

Vorlesung vom 29.04.2014
Rest Kapitel 3 + Kapitel 4 + Kapitel 5:
Ellipse + Füllen + Dithering + Clipping

Ellipse um (0,0) parametrisiert



$$x = a \cdot \cos(\alpha)$$

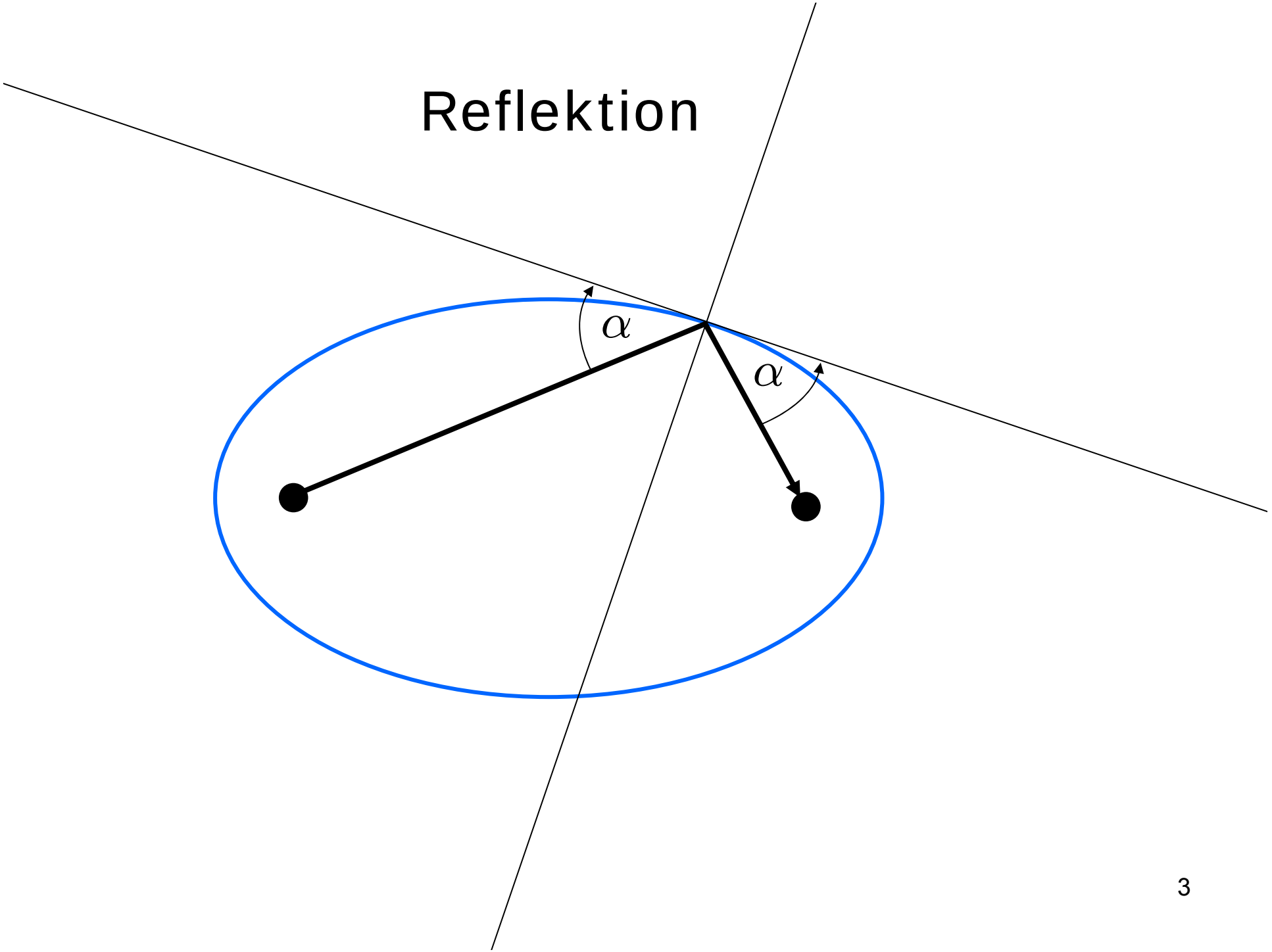
$$y = b \cdot \sin(\alpha)$$

$$U \approx 2\pi\sqrt{\tfrac{1}{2}(a^2 + b^2)}$$

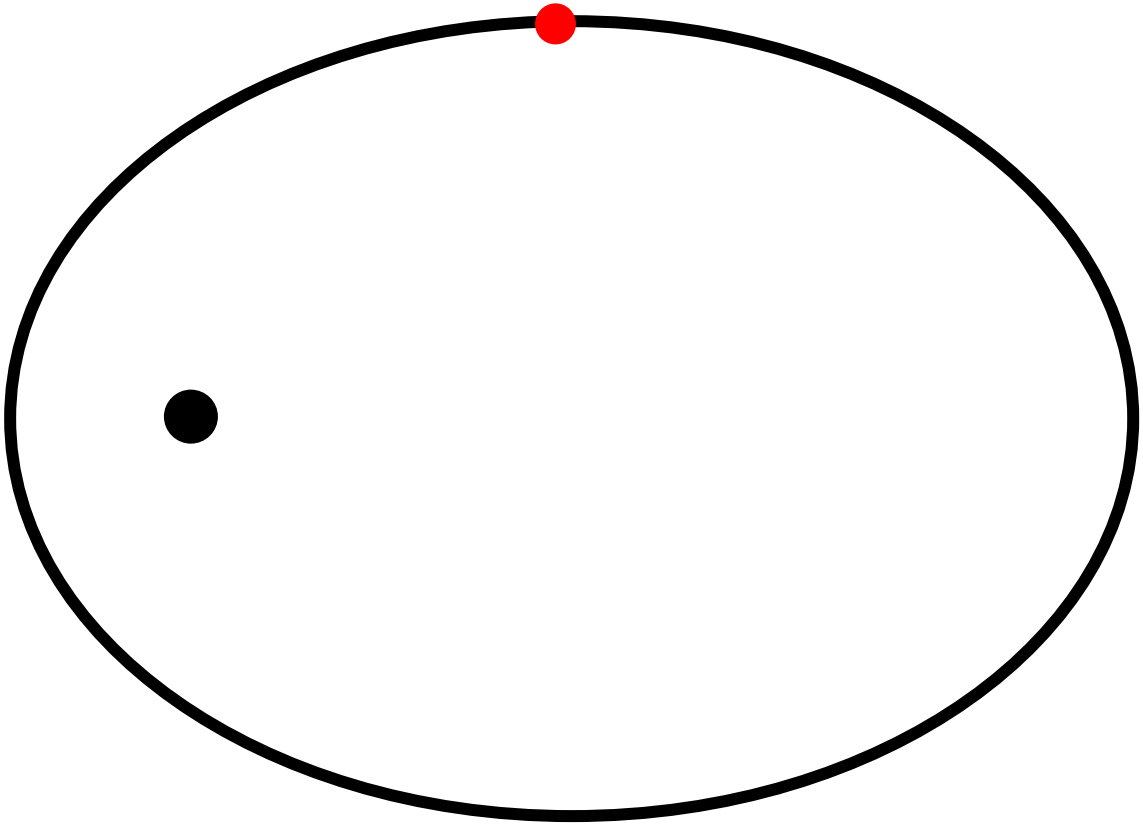
$$A = \pi a b$$

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

Reflektion

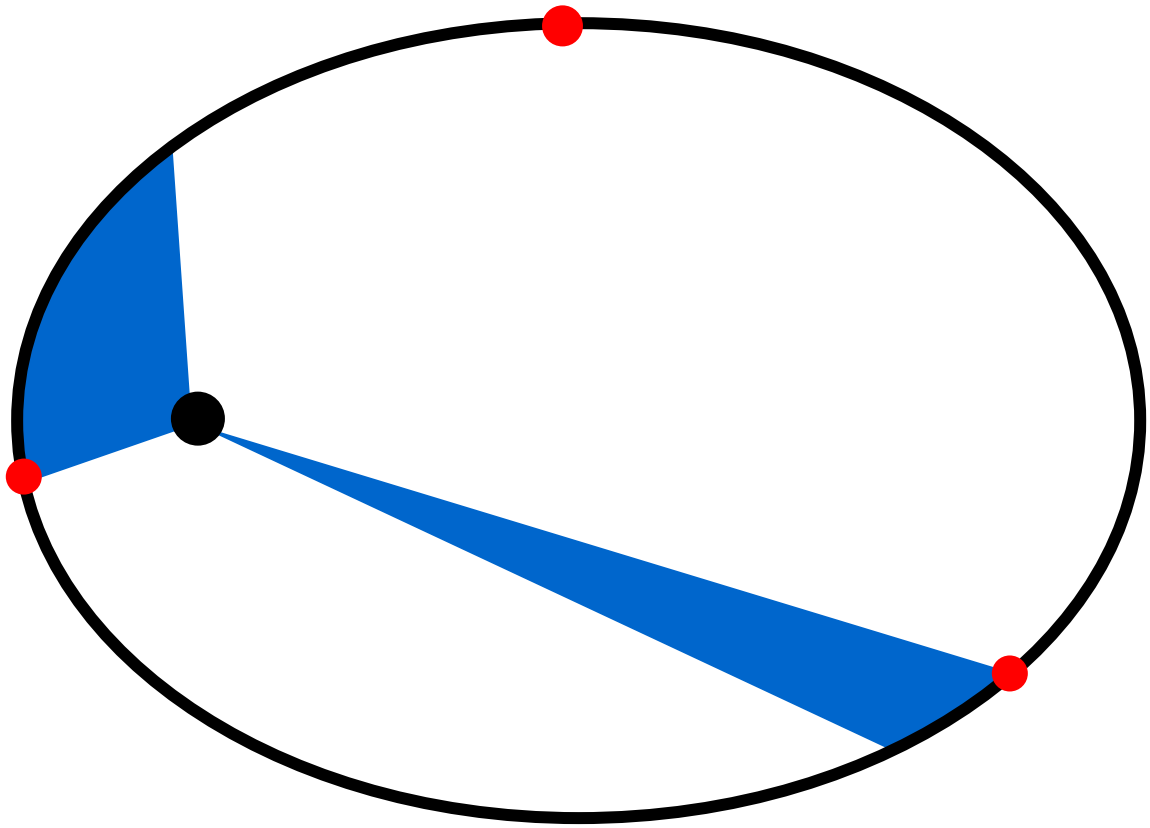


1. Keplersches Gesetz



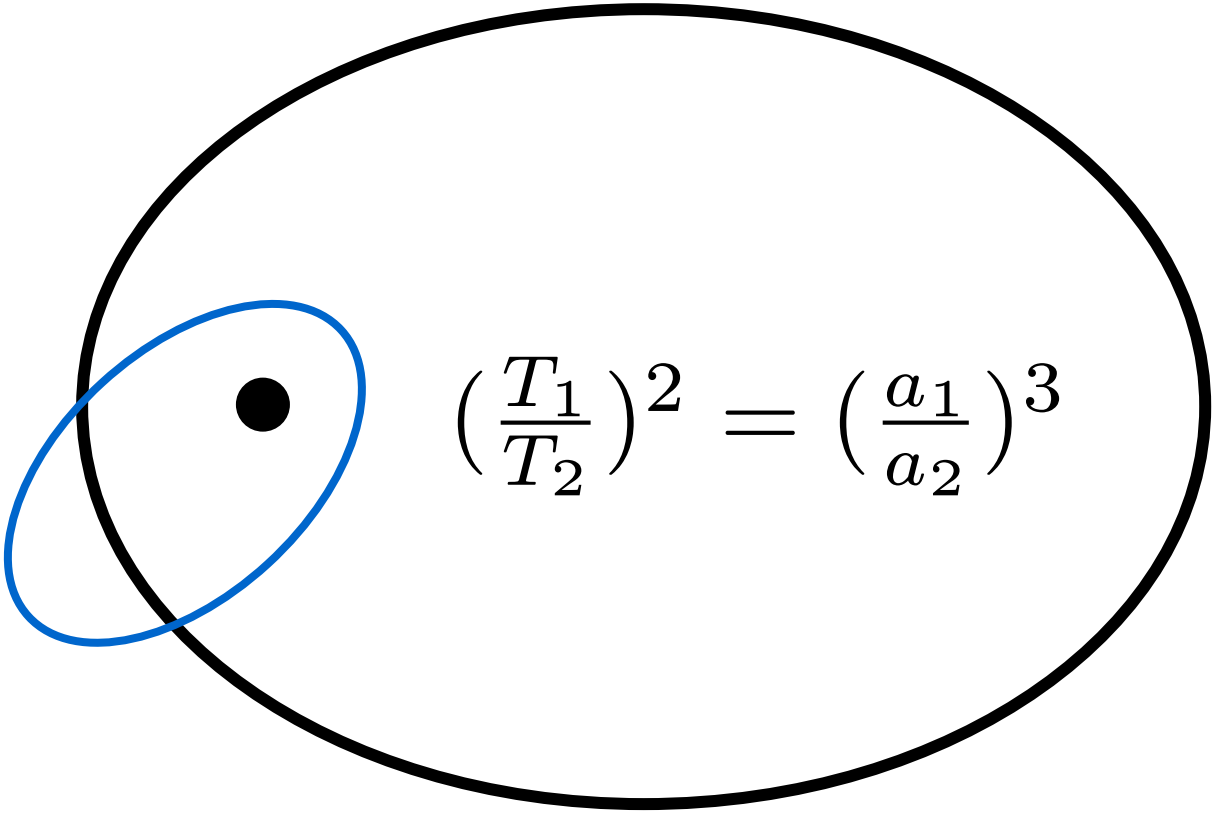
Die Planeten umkreisen die Sonne auf einer Ellipse

2. Keplersches Gesetz



In gleichen Zeiten überstreicht der Fahrstrahl gleiche Flächen

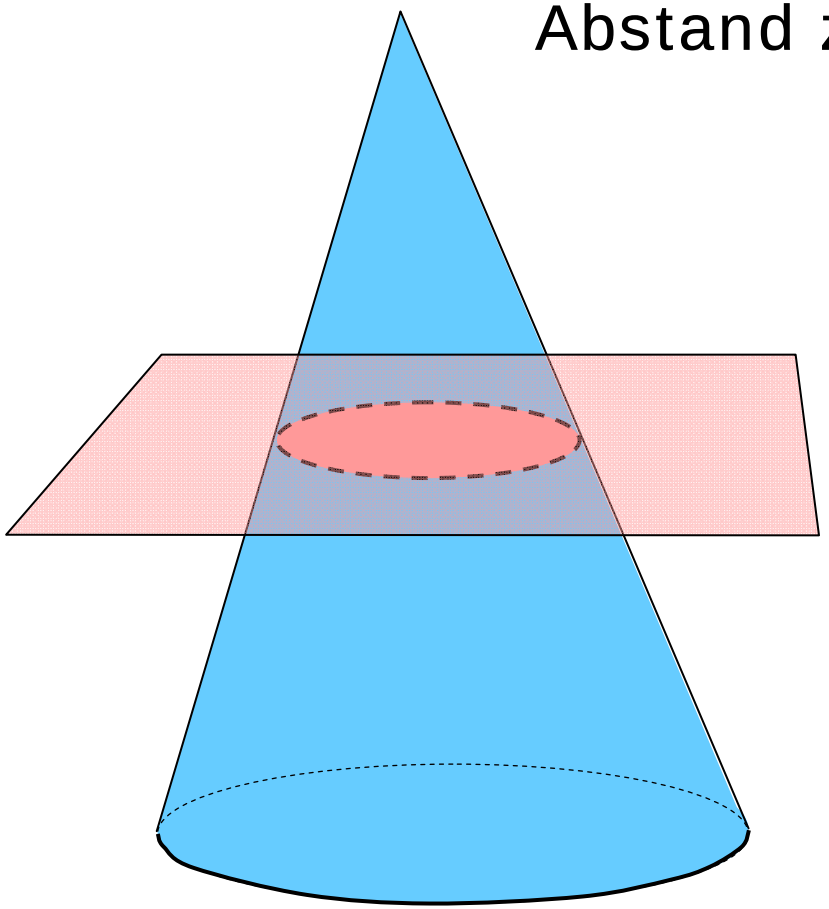
3. Keplersches Gesetz



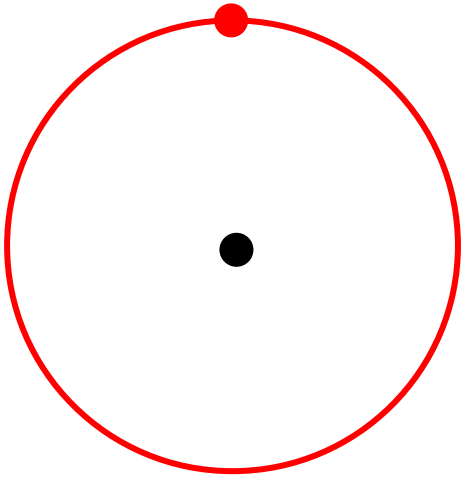
Die Quadrate der Umlaufzeiten verhalten sich
wie die Kuben der großen Halbachsen

Kegelschnitt: Kreis

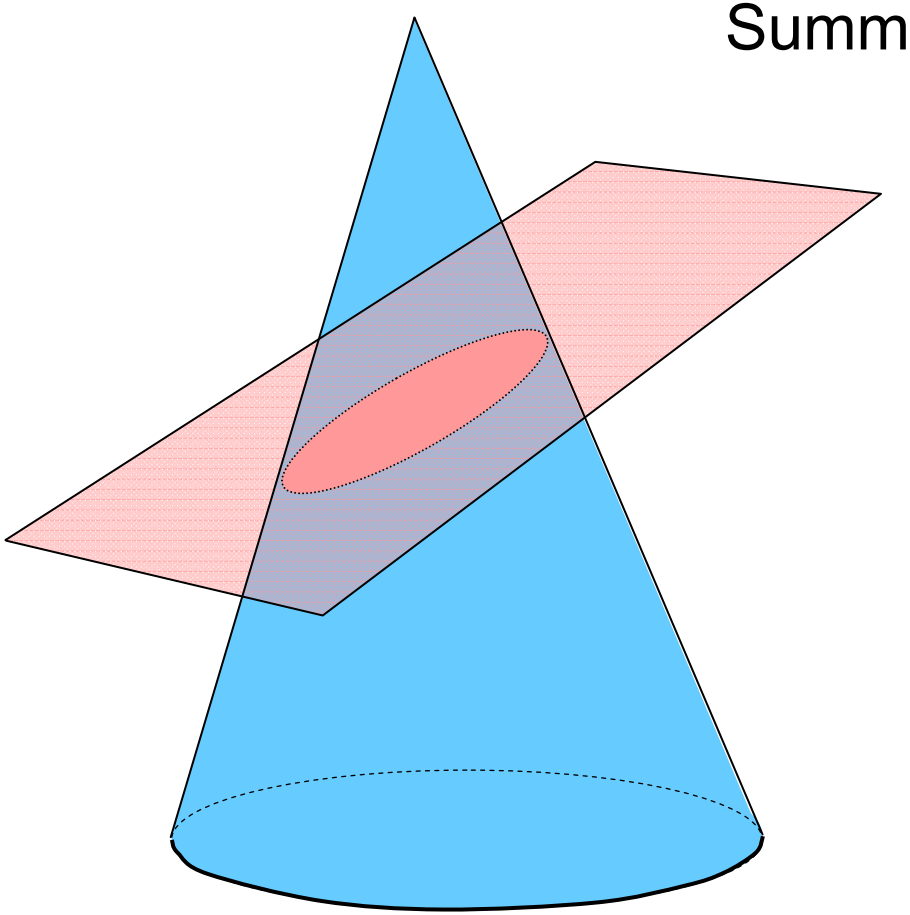
Abstand zu einem Punkt ist konstant



$$x^2 + y^2 = 1$$

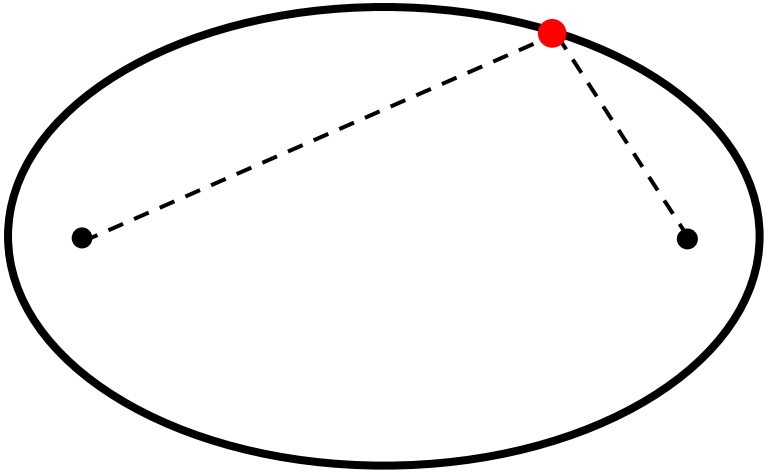


Kegelschnitt: Ellipse

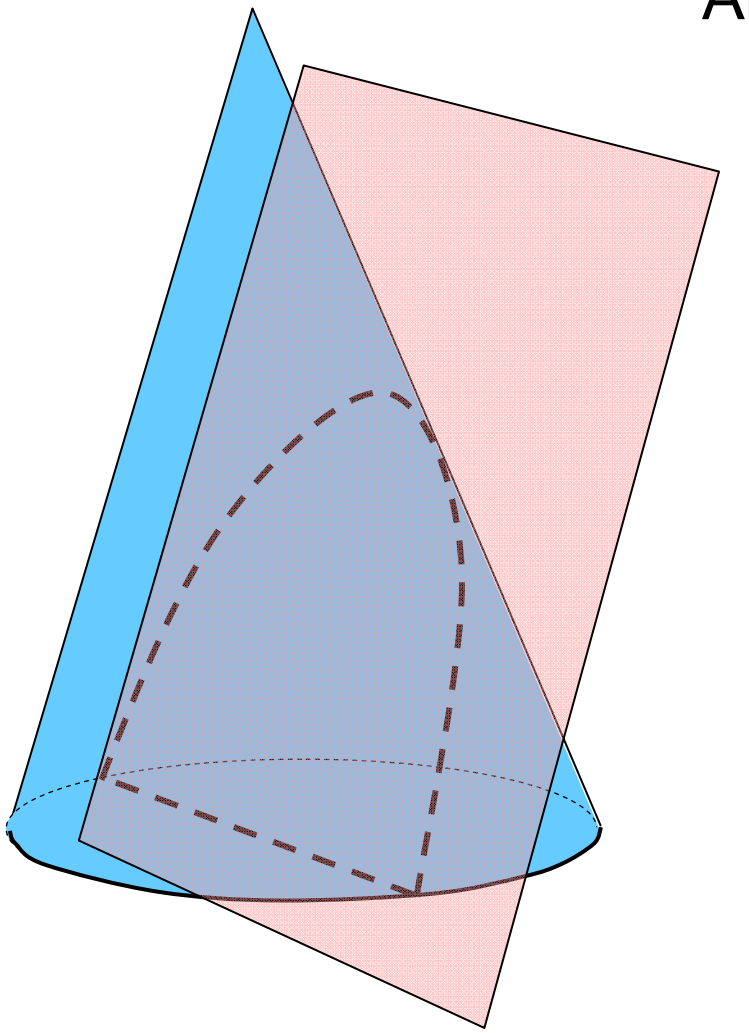


Summe der Abstände zu 2 Punkten ist konstant

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

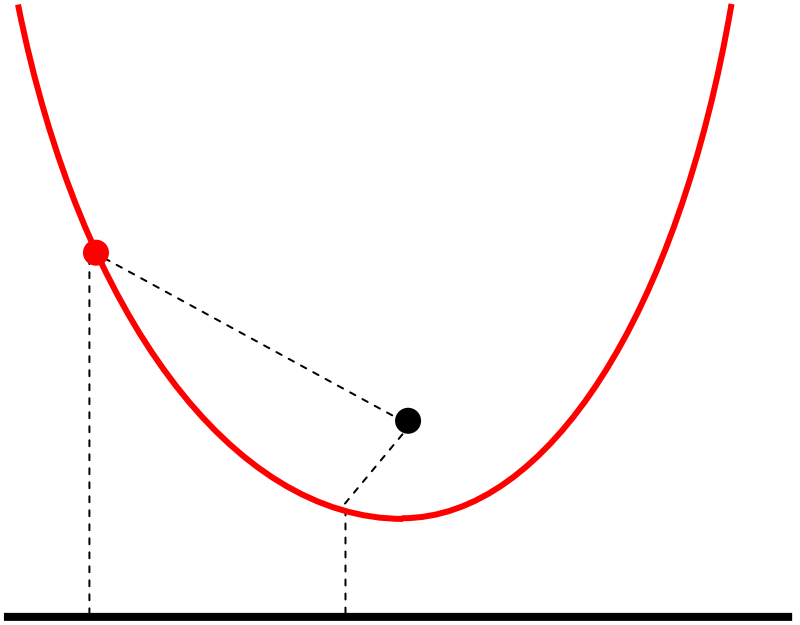


Kegelschnitt: Parabel



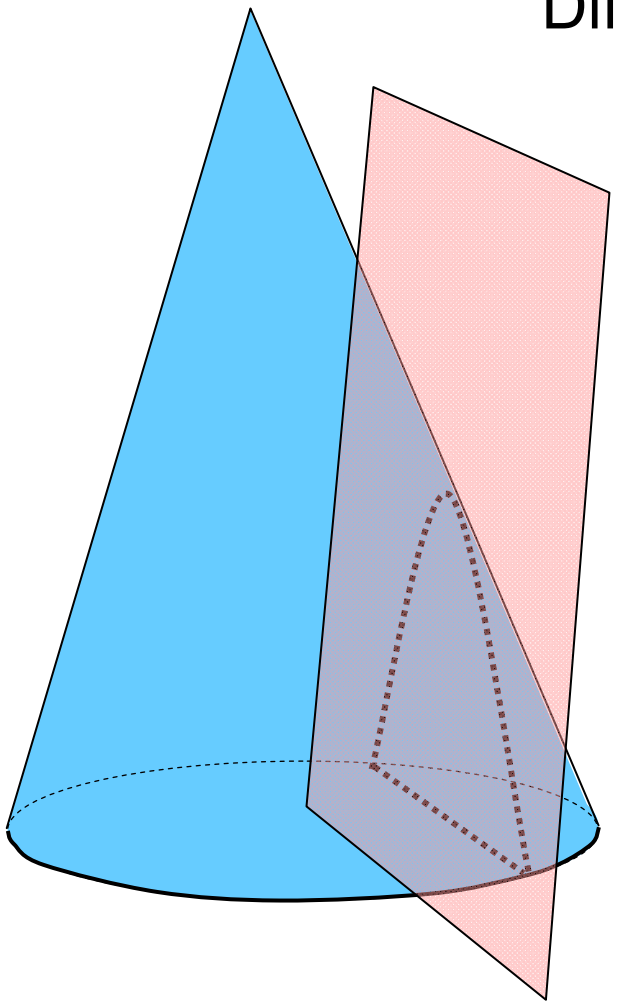
Abstand zu Punkt und Gerade ist gleich

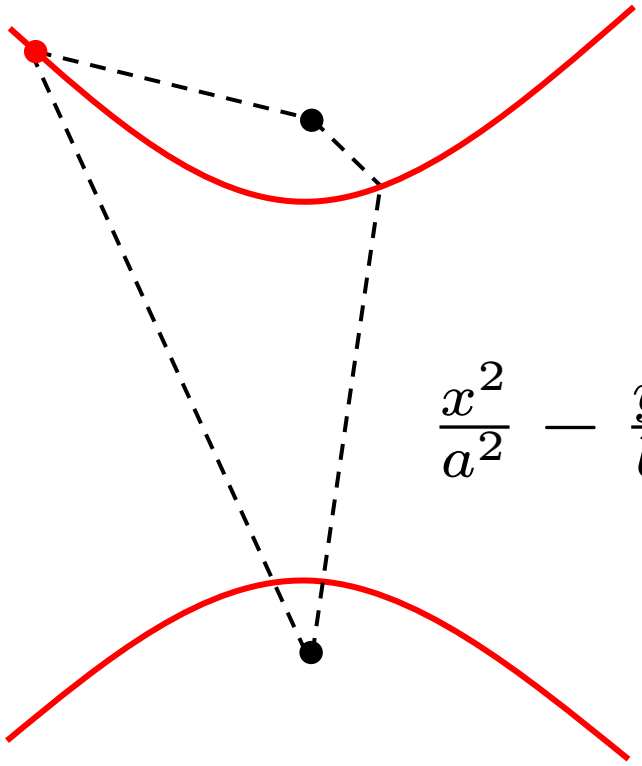
$$y = ax^2 + bx + c$$



Kegelschnitt: Hyperbel

Differenz der Abstände zu 2 Punkten ist konstant





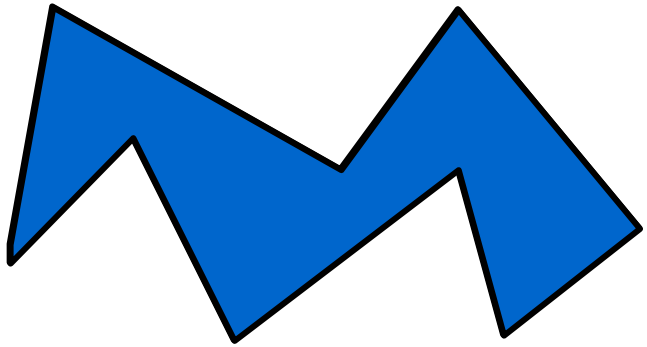
$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$$

Computergrafik SS 2014

Oliver Vornberger

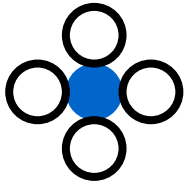
Kapitel 4:
2D-Füllen

Füllverfahren

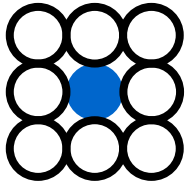


- Universelle Füllverfahren
  (Zusammenhangseigenschaften)
- Scan-Line-Verfahren
  (Geometrie)

Universelle Füllverfahren



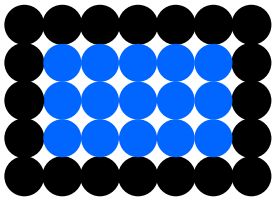
4-way-stepping



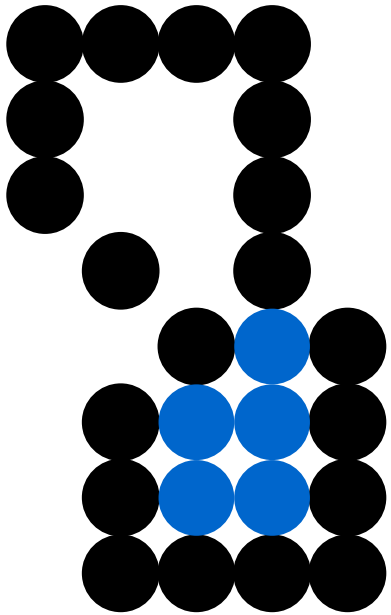
8-way-stepping

Beginnend beim Saatpixel:

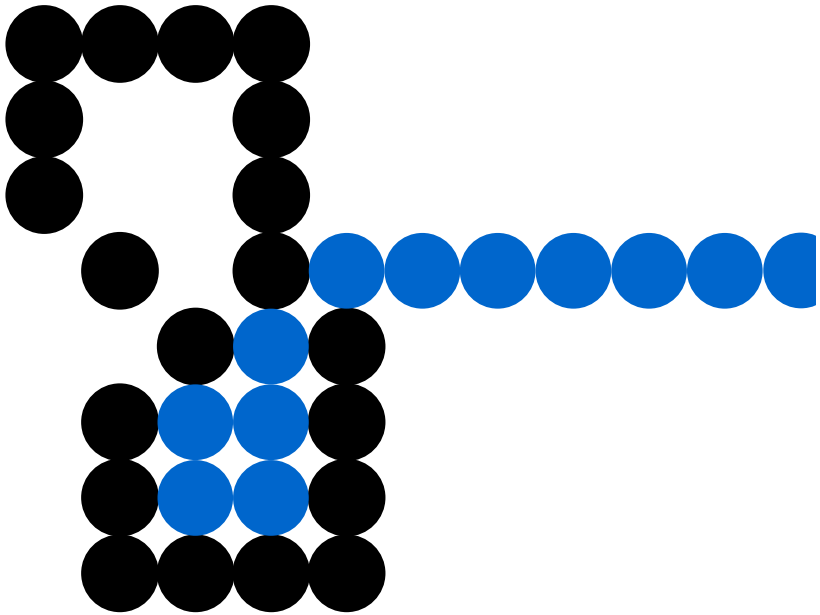
färbe alle Nachbarn, bis Umgrenzung erreicht ist.



Probleme beim universellen Füllen



4-way-stepping



8-way-stepping

Rekursives Füllen

`boolean rangeOK(x,y)` true, falls Punkt x,y innerhalb des Bildbereichs

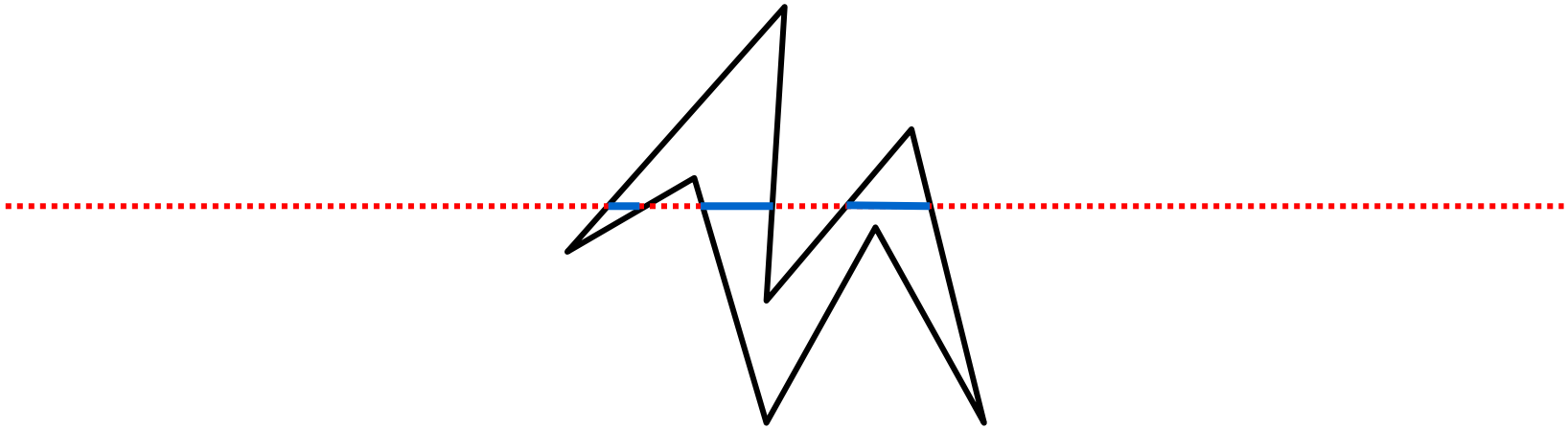
`boolean getPixel(x,y)` true, falls Vordergrundfarbe an Punkt x,y

`void setPixel(x,y)` setze Vordergrundfarbe an Punkt x,y

```
public void boundaryFill(int x, int y){
if (rangeOk(x,y) && !getPixel(x,y)){
  setPixel(x,y);
  boundaryFill(x+1,y);
  boundaryFill(x,  y+1);
  boundaryFill(x-1,y);
  boundaryFill(x,  y-1);
}
```

~cg/2014/skript/Applets/2D-basic/App.html

Scan-Line-Verfahren



Bewege waagerechte Scan-Line von oben nach unten über das Polygon und färbe entsprechende Abschnittsgeraden

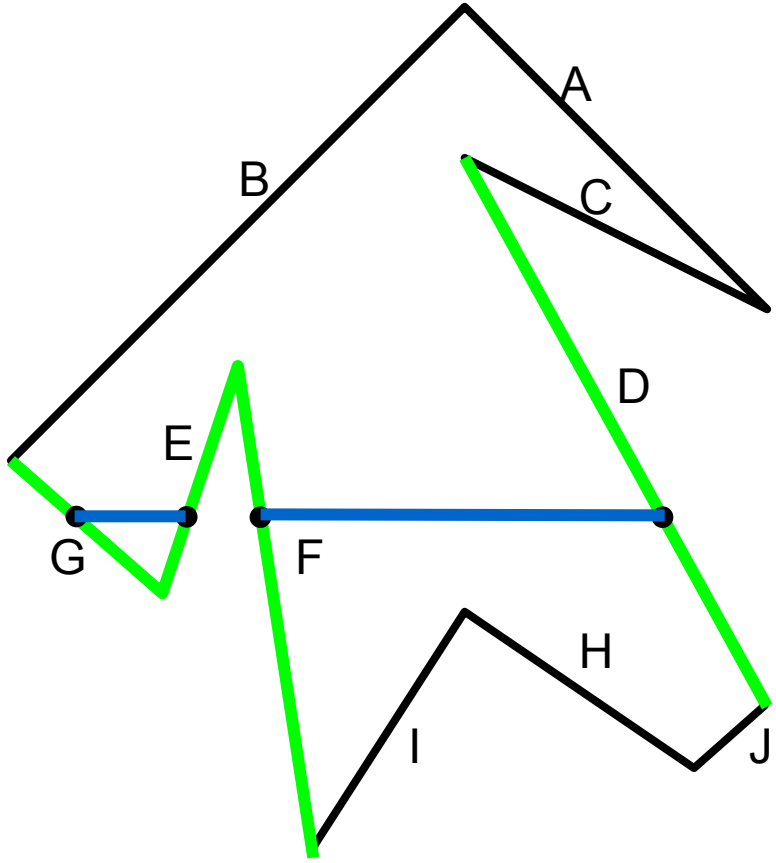
Scan-Line-Verfahren: Detail

1. Sortiere Kanten nach größtem y-Wert

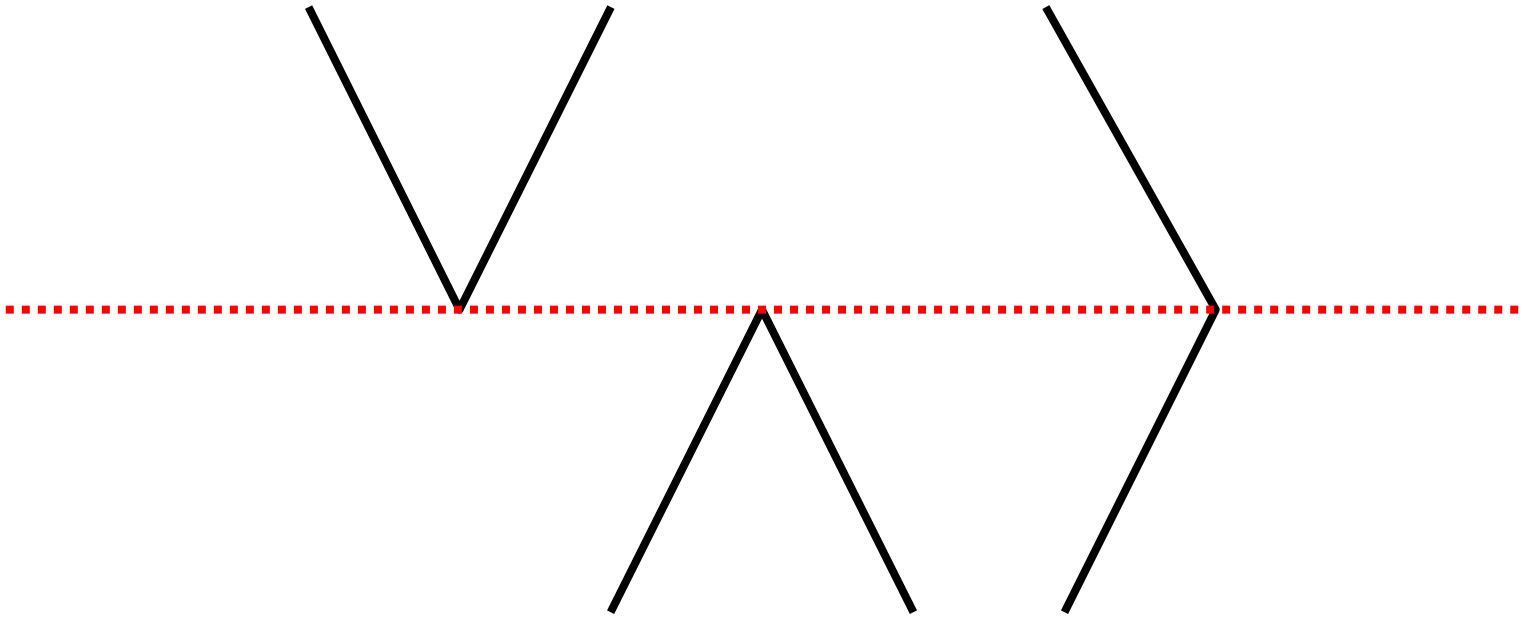
2. Bewege Scan-Line von oben nach unten

3. für jede Position der Scan-Line:
   ermittele aktive Kanten
   berechne Schnittpunkte mit Scan-Line
   sortiere die Schnittpunkte nach x Werten
   färbe abwechselnd zwischen Schnittpunkten

Scan-Line-Verfahren: Beispiel

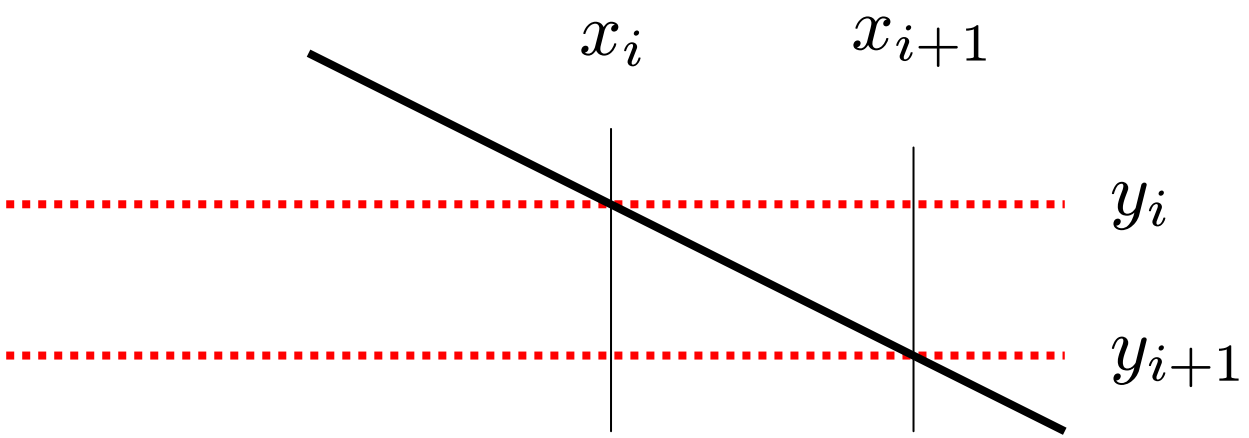


Problemfälle



doppelt doppelt einfach

Schnittpunkte fortschreiben



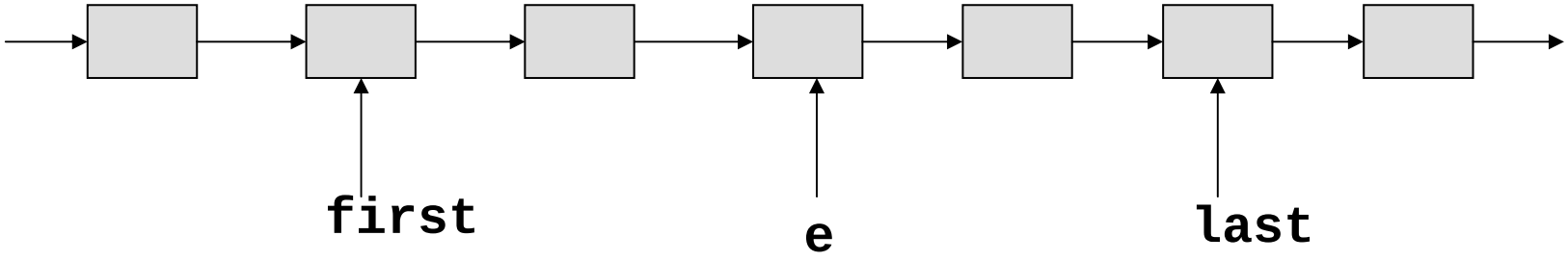
$$s = \frac{y_i - y_{i+1}}{x_i - x_{i+1}} \qquad y_i - y_{i+1} = 1$$

$$x_i - x_{i+1} = \frac{y_i - y_{i+1}}{s} \qquad x_{i+1} = x_i - \frac{1}{s}$$

~cg/2014/skript/Applets/2D-basic/App.html

Datenstruktur für Kante

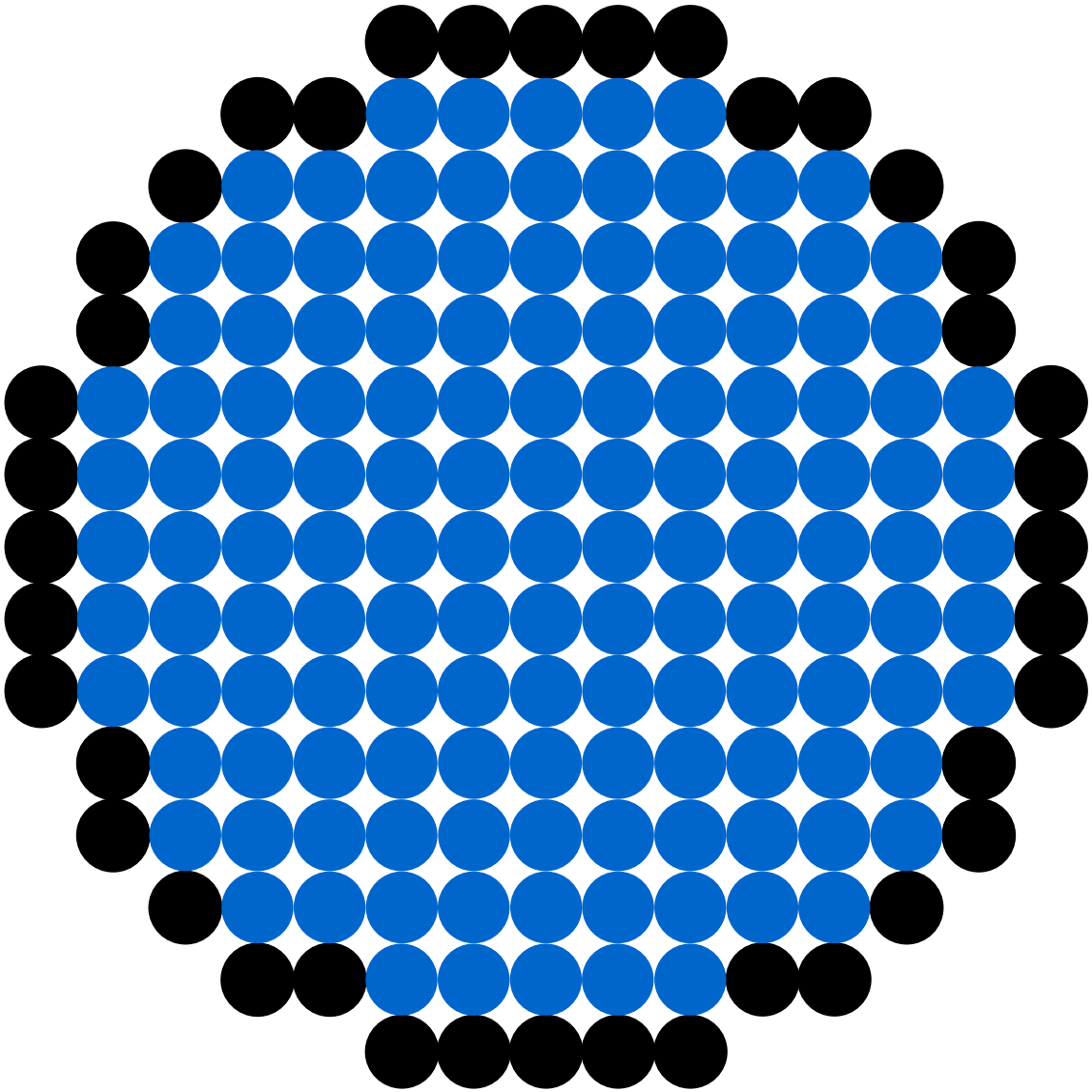
```
public class Edge {
       int y_top;       // groesster y-Wert
       int delta_y;     // Ausdehnung in y-Richtung
       double delta_x; // inverse Steigung
       double x_int;    // errechneter Schnittpunkt
       Edge next;       // Verweis auf naechste Kante
}
```



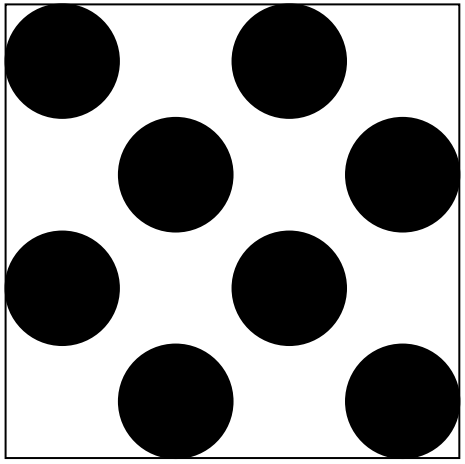

```
if (e.delta_y) > 0) {
  e.delta_y--;
  e.x_int = e.x_int – e.delta_x;
  e = e.next;
}
```

~cg/2014/skript/Applets/2D-basic/App.html

Scan-Line-Verfahren für Kreis



Dither-Matrix: Definition

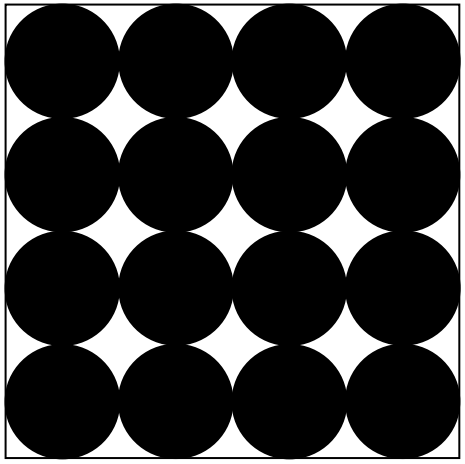


0	8	2	10
12	4	14	6
3	11	1	9
15	7	13	5

Eine *n x n* Dithermatrix enthält gleichmäßig verteilt alle Zahlen aus dem Intervall [$0..n^2 - 1$]

Für Grauwert $0 \leq k \leq n^2$
setze alle Pixel mit Eintrag < k

Dither-Matrix: Beispiel



0	8	2	10
12	4	14	6
3	11	1	9
15	7	13	5

Dither-Matrix: Konstruktion

$$D_0 = (0)$$

U_n = $n \times n$ - Matrix, besetzt mit 1

$$D_n = \begin{pmatrix} 4 \cdot D_{n-1} + 0 \cdot U_{n-1} & 4 \cdot D_{n-1} + 2 \cdot U_{n-1} \\ 4 \cdot D_{n-1} + 3 \cdot U_{n-1} & 4 \cdot D_{n-1} + 1 \cdot U_{n-1} \end{pmatrix}$$

$$D_1 = \begin{pmatrix} 0 & 2 \\ 3 & 1 \end{pmatrix} \qquad D_2 = \begin{pmatrix} 0 & 8 & 2 & 10 \\ 12 & 4 & 14 & 6 \\ 3 & 11 & 1 & 9 \\ 15 & 7 & 13 & 5 \end{pmatrix}$$

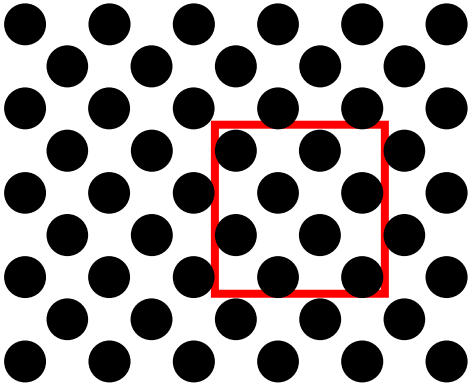
$$\begin{pmatrix} 0 & 8 & 0 & 8 \\ 12 & 4 & 12 & 4 \\ 0 & 8 & 0 & 8 \\ 12 & 4 & 12 & 4 \end{pmatrix} \quad \begin{pmatrix} 0 & 0 & 2 & 2 \\ 0 & 0 & 2 & 2 \\ 3 & 3 & 1 & 1 \\ 3 & 3 & 1 & 1 \end{pmatrix}$$

Dither-Matrix: Aufruf

Gegeben N × N Dithermatrix D.

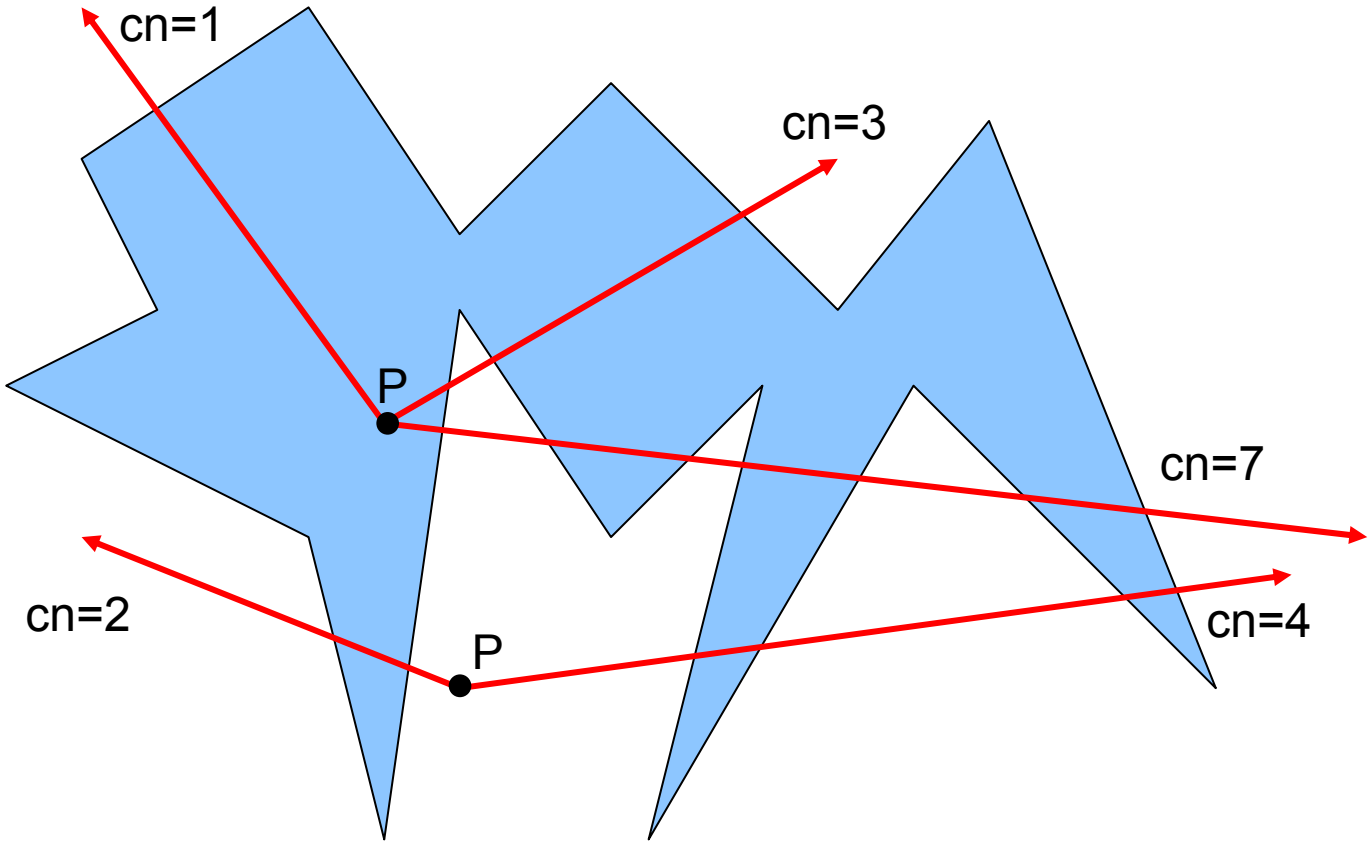
Einfärben an Position (x,y) mit Grauwert k:

```
if (D[x%N][y%N] < k)
    setPixel(x,y);
else
    delPixel(x,y);
```

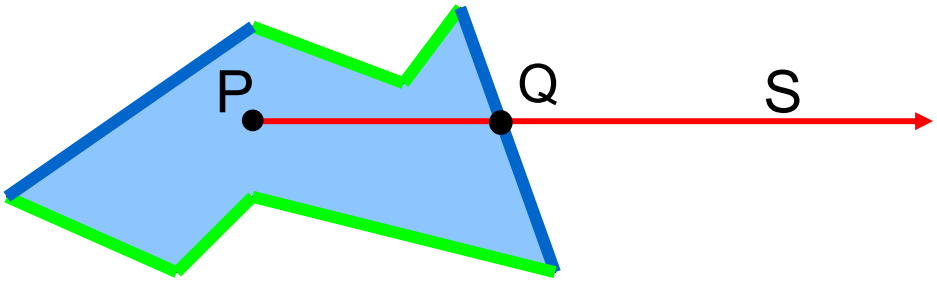


~cg/2014/skript/Applets/2D-basic/App.html

Punkt in Polygon



Kreuzungszahl berechnen



Sei *S* der von *P* nach rechts gehende Strahl

Für jede Polygonkante von P_1 nach P_2:

falls P_1 und P_2 oberhalb: kein Schnittpunkt

falls P_1 und P_2 unterhalb: kein Schnittpunkt

falls P_1 und P_2 auf verschiedenen Seiten:

berechne Schnittpunkt *Q* mit S

falls rechts von *P*: erhöhe Kreuzungszahl

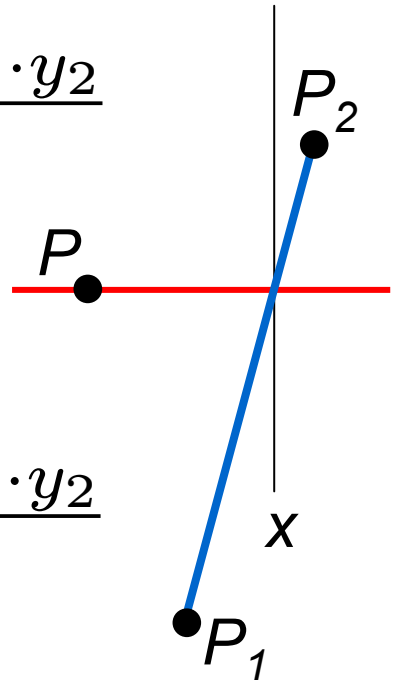
Schnittpunkt berechnen

$$f(x) = \frac{y_2 - y_1}{x_2 - x_1} \cdot x + \frac{x_2 \cdot y_1 - x_1 \cdot y_2}{x_2 - x_1}$$

$$f(x) = y$$

$$y = \frac{y_2 - y_1}{x_2 - x_1} \cdot x + \frac{x_2 \cdot y_1 - x_1 \cdot y_2}{x_2 - x_1}$$

$$x = \frac{y \cdot (x_2 - x_1) - x_2 \cdot y_1 + x_1 \cdot y_2}{y_2 - y_1}$$



public boolean contains (int x, int y)

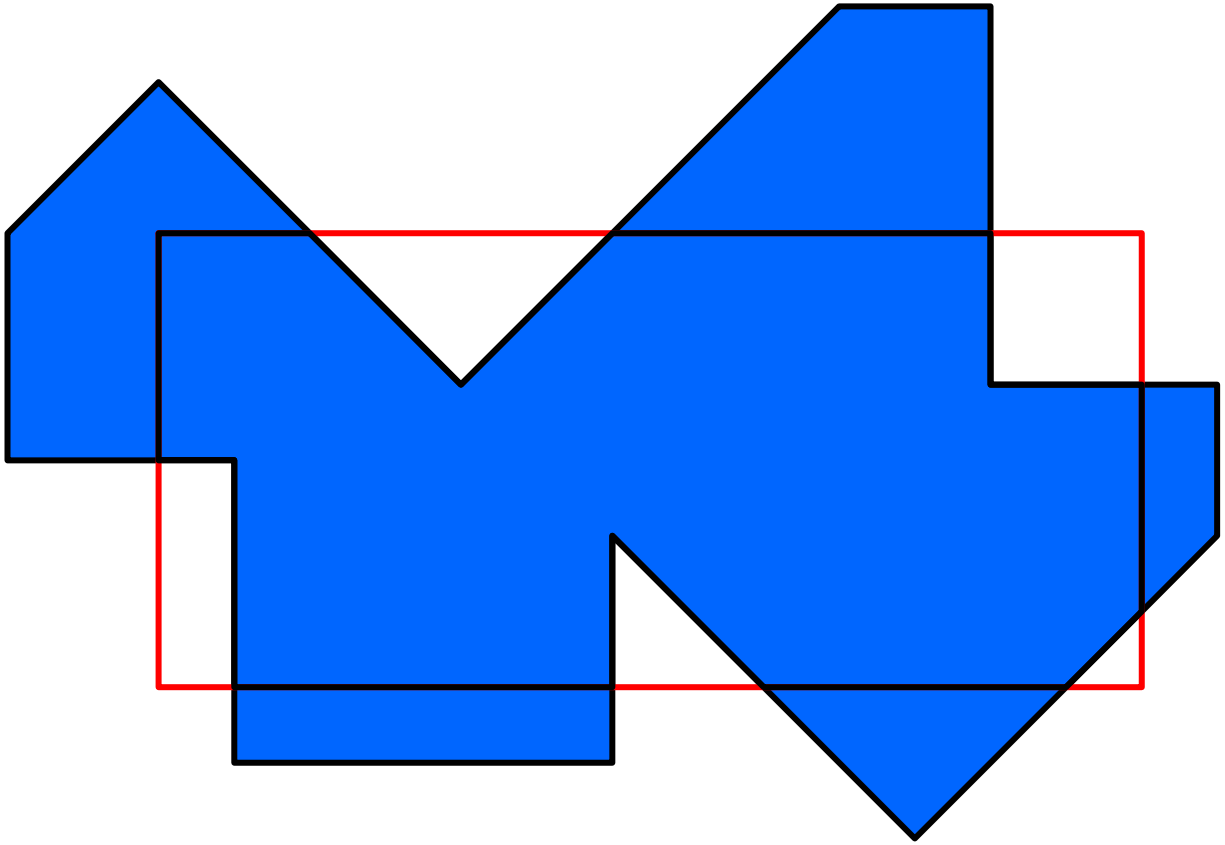
```
x1 = xpoints[n-1]; y1 = ypoints[n-1];
x2 = xpoints[0];   y2 = ypoints[0];
boolean inside = false;
boolean startUeber = y1 >= y ? true : false;
for (i=1; i<n; i++) {
  boolean endUeber = y2 >= y ? true : false;
  if ((startUeber != endUeber &&
   (double)(y*(x2-x1)- x2*y1 + x1*y2)/(y2-y1)>=x))
      inside = !inside;
  startUeber = endUeber;
  y1=y2; x1=x2; x2=xpoints[i]; y2=ypoints[i];
}
return inside;
```

Computergrafik SS 2014

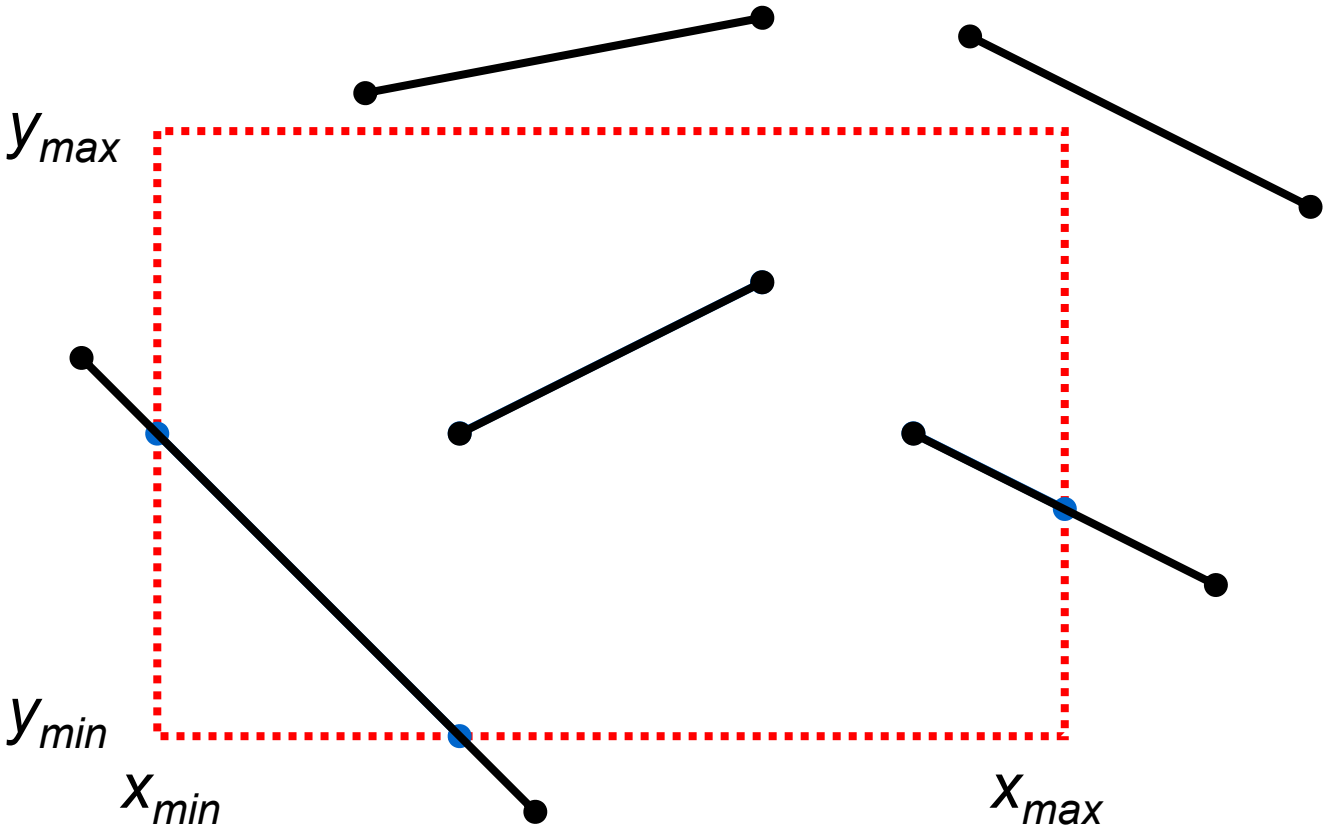
Oliver Vornberger

Kapitel 5:
2D-Clipping

2D-Clipping



Clipping von Linien



Region Code: Definition

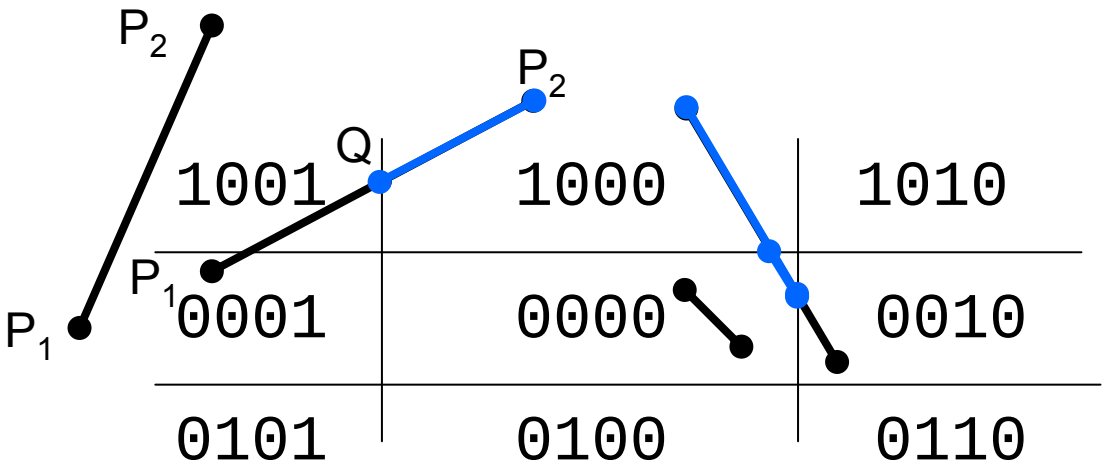
Bit 0: links Bit1:rechts Bit2: unten Bit3: oben

1001	**1000**	**1010**
y_{max}		
0001	**0000**	**0010**
y_{min}		
0101	x_{min} **0100**	x_{max} **0110**

Region Code: Berechnung

```
private static final byte CENTER = 0;
private static final byte LEFT   = 1;
private static final byte RIGHT  = 2;
private static final byte BOTTOM = 4;
private static final byte TOP    = 8;
public byte region_code (int x, int y) {
  byte c = CENTER;
  if (x < xmin) c = LEFT;
  if (x > xmax) c = RIGHT;
  if (y < ymin) c = c | BOTTOM;
  if (y > ymax) c = c | TOP;
  return c;
}
```

Cohen & Sutherland



falls code(P_1) && code(P_2) ≠ 0 $\Rightarrow$ komplett außerhalb

falls code(P_1) || code(P_2) = 0 $\Rightarrow$ komplett innerhalb

sonst: berechne Schnittpunkt Q und teste Restlinie erneut

Quiz

P1 und P2 liegen komplett außerhalb, falls

1001	1000	1010
0001	0000	0010
0101	0100	0110

A Code(P1) || code(P2) = 0
| 0,0 %

B Code(P1) || code(P2) ≠ 0
| 0,0 %

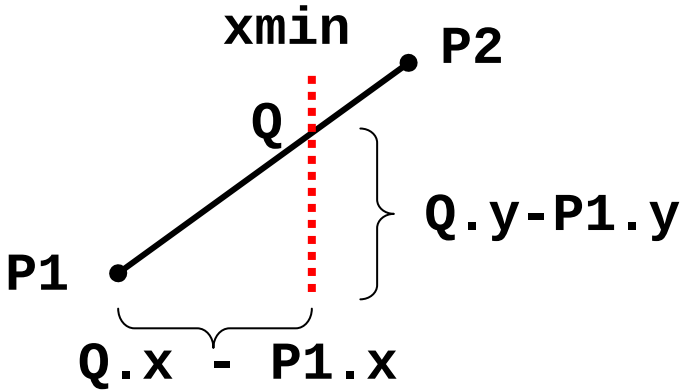
C Code(P1) && code(P2) = 0
| 0,0 %

D Code(P1) && code(P2) ≠ 0
| 0,0 %

Anzahl der Stimmen: 0

Abstimmung läuft ...

Schnittpunkte



$$slope = \frac{Q.y - P1.y}{Q.x - P1.x}$$

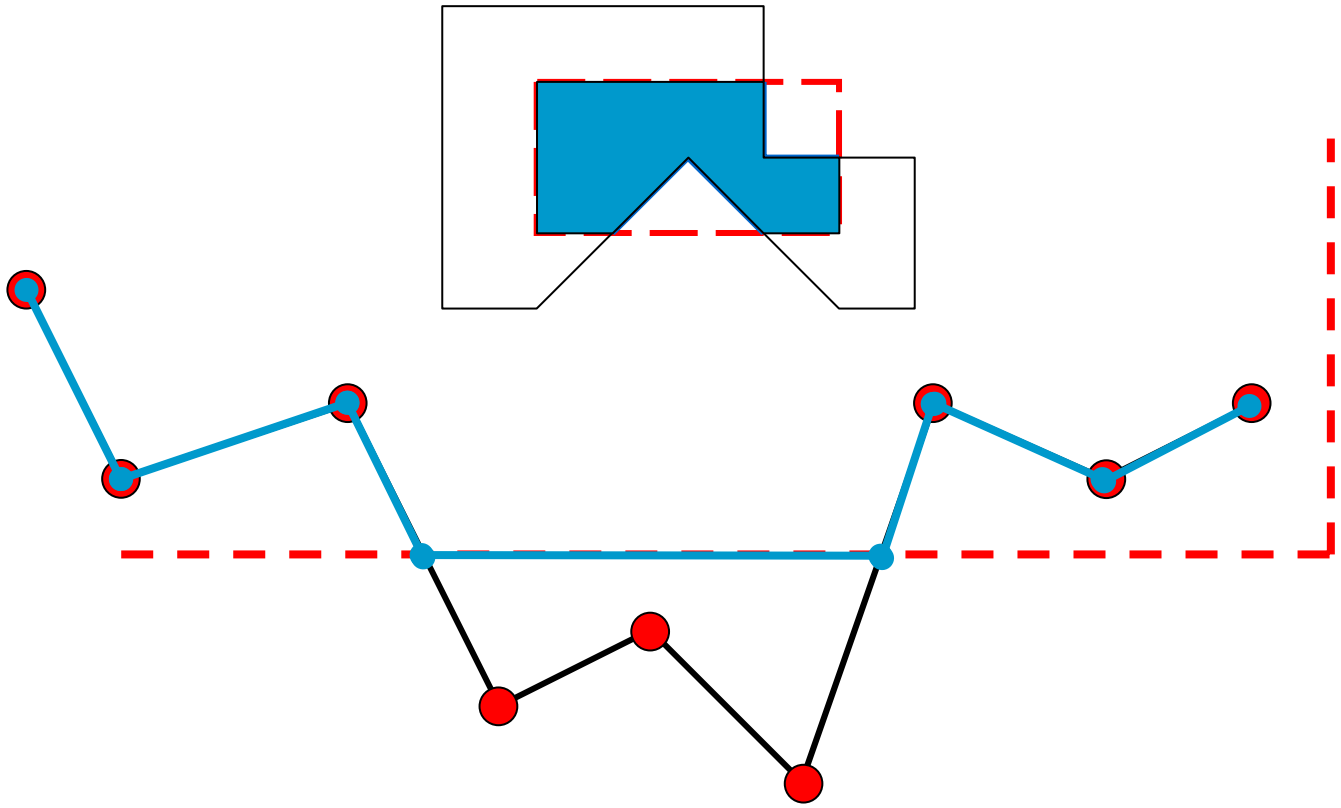
$$slope = \frac{P2.y - P1.y}{P2.x - P1.x}$$

```
slope = (double)(P2.y - P1.y)/(P2.x - P1.x);
Q.x   = xmin
Q.y   = (int)(Q.x-P1.x)*slope + P1.y
```

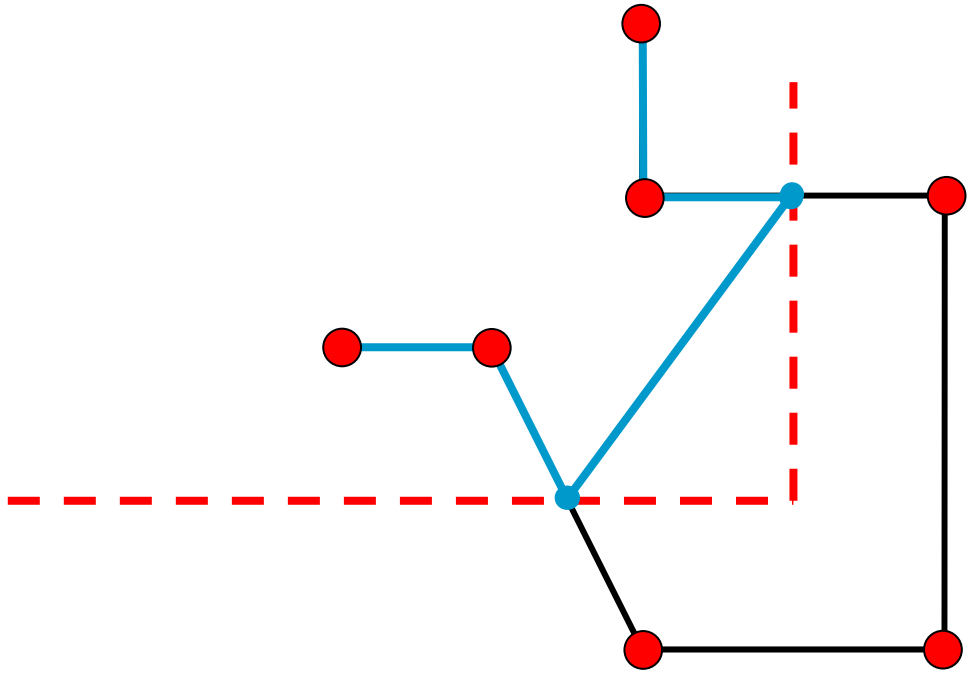
Cohen-Sutherland

```
boolean cohen_sutherland(Point P1,
                         Point P2,
                         Point Q1,
                         Point Q2){
// clippt Gerade P1,P2 am Fenster
// liefert true, falls sichtbar
// liefert in Q1,Q2 den sichtbaren Teil
...
}
```

Clipping von Polygonen



Problem bei Clip-Fenster-Ecken



Sutherland & Hodgman

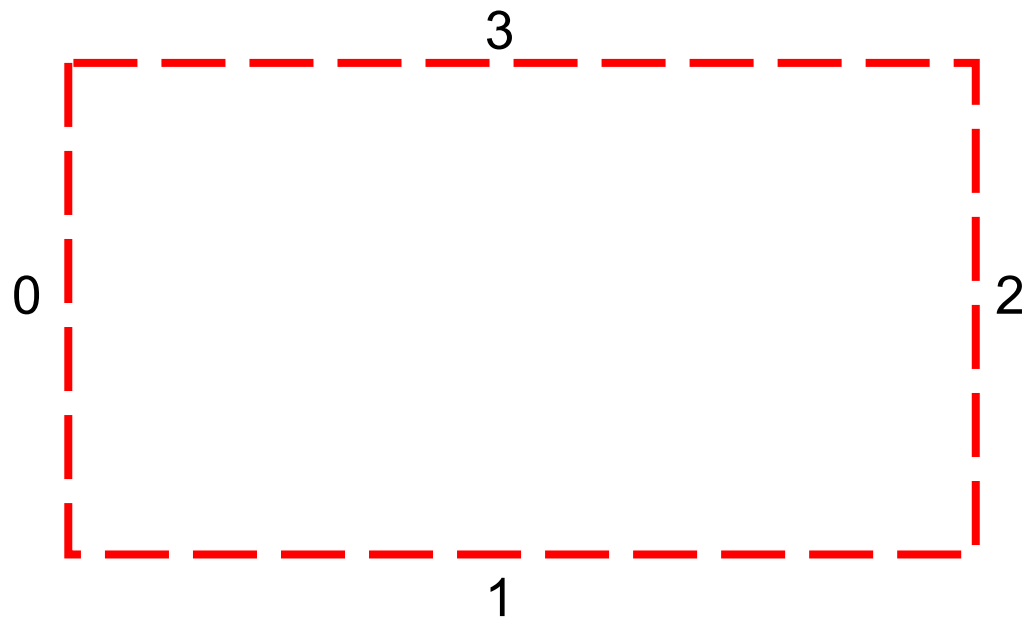
für eine Clipping-Gerade E

für jeden Polygonpunkt P_i:

falls P_i sichtbar: übernimm P_i

falls Kante von P_i zu P_{i+1} E schneidet:
übernimm Schnittpunkt

4 Clipping-Kanten



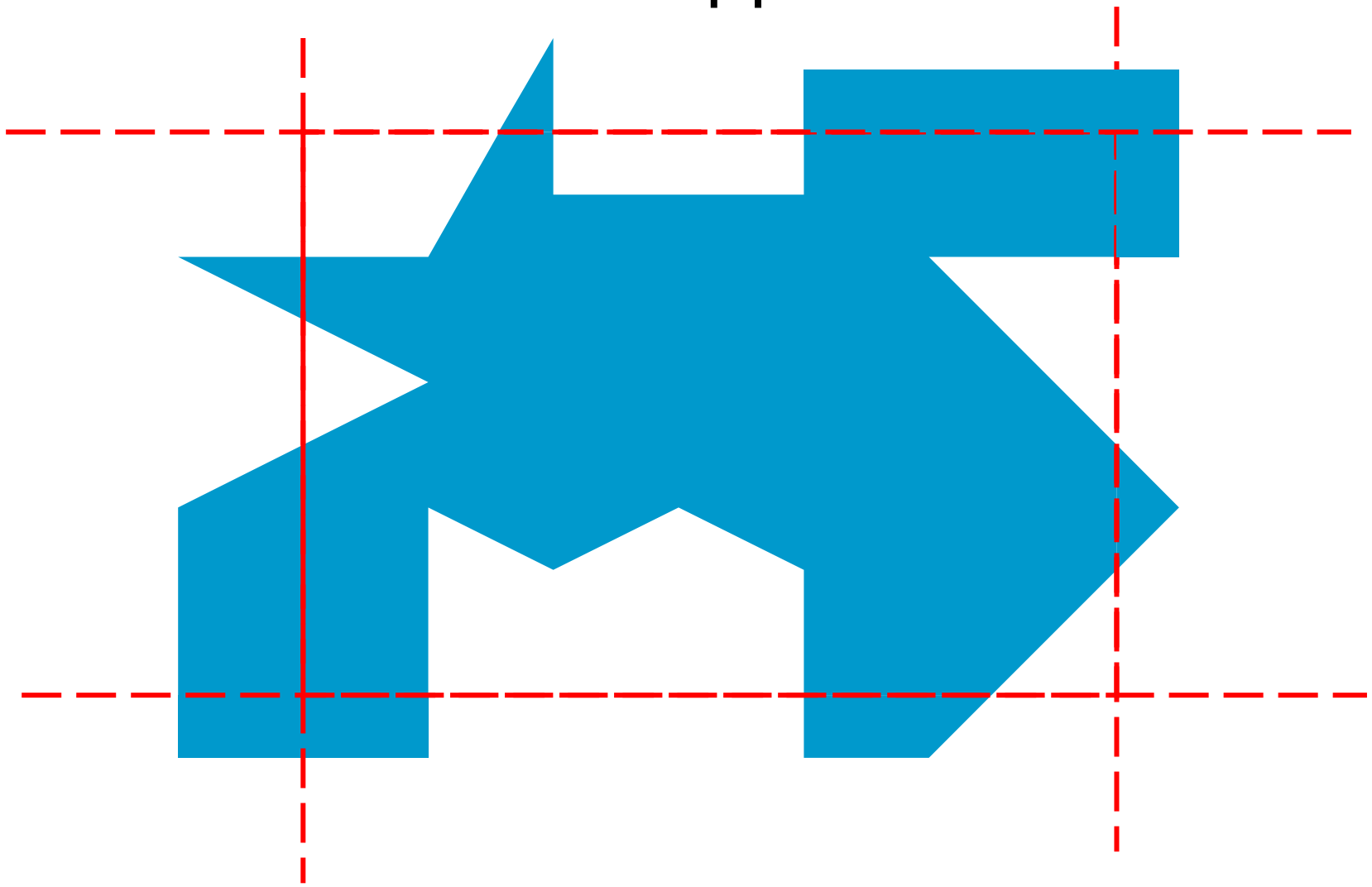
Sichtbarkeitstest

```
boolean On_Visible_Side(
            Point P, int wert, int fall) {
  switch (fall) {
    case 0: return (P.x >= wert);
    case 1: return (P.y >= wert);
    case 2: return (P.x <= wert);
    case 3: return (P.y <= wert);
  }
}
```

Schnittpunkt

```
boolean intersection(
    Point P1, Point P2,
    int wert, int fall, Point I) {
...
P1_vis = On_Visible_Side(P1,wert,fall);
P2_vis = On_Visible_Side(P2,wert,fall);
...
slope =(double)(P2.y-P1.y)/
       (double)(P2.x-P1.x);
if (fall %2 == 0) {
    I.x = (int) wert;
    I.y = (int)(wert-P1.x)*slope + P1.y;
...
```

4 x Clippen



~cg/2014/skript/Applets/2D-basic/App.html