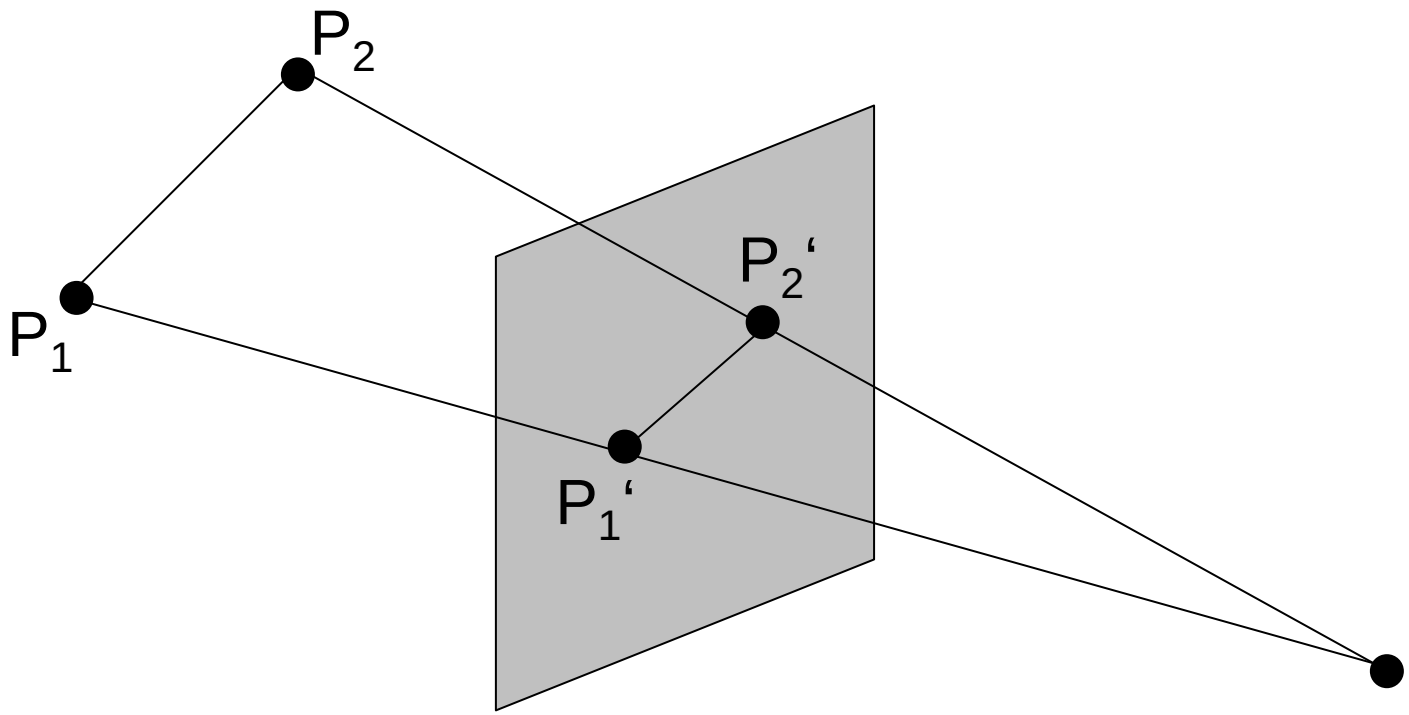
Computergrafik SS 2014

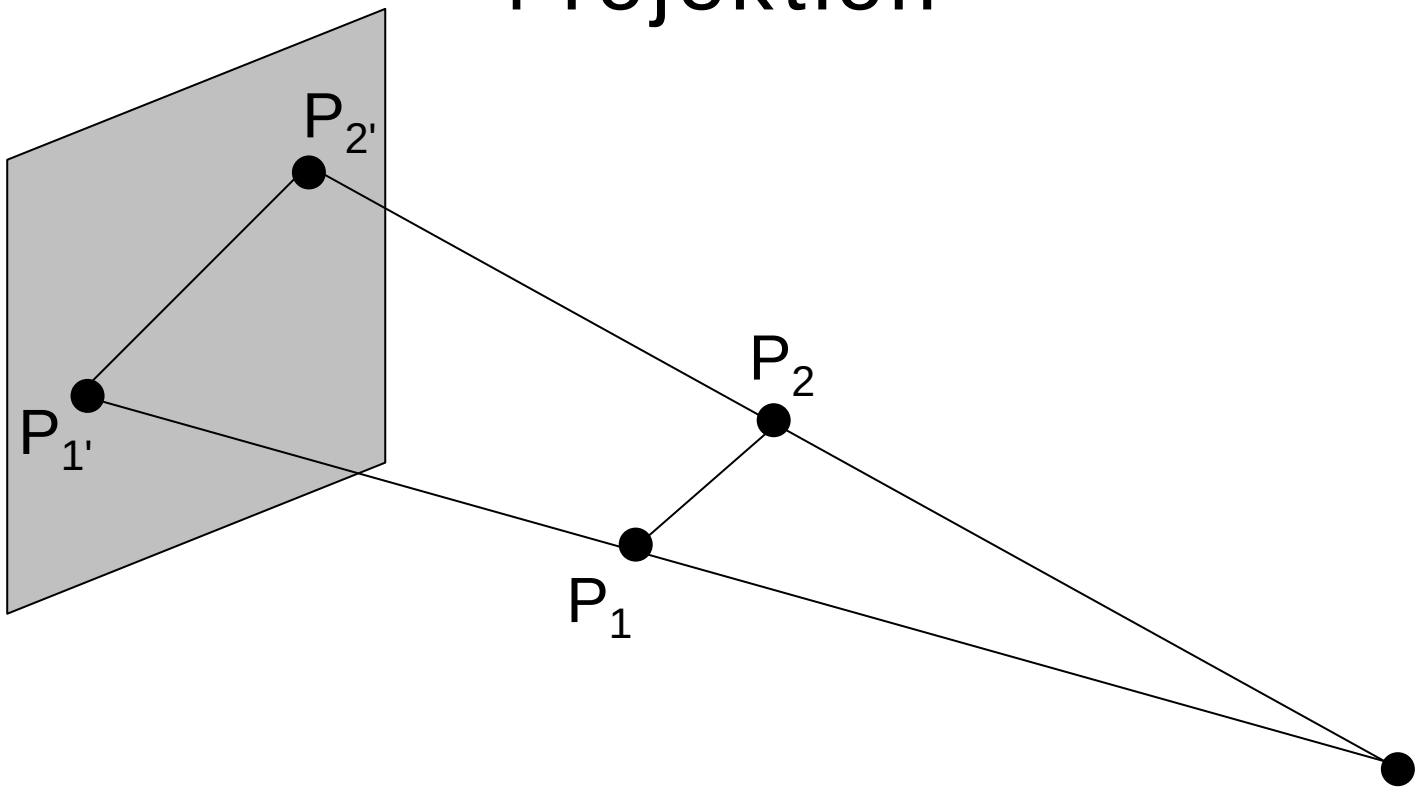
Oliver Vornberger

Kapitel 14:
Projektion

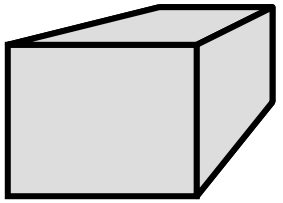
Projektion



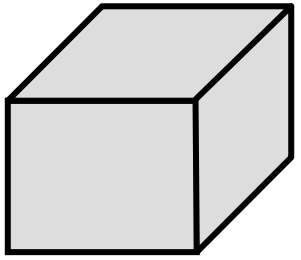
Projektion



Projektionsarten

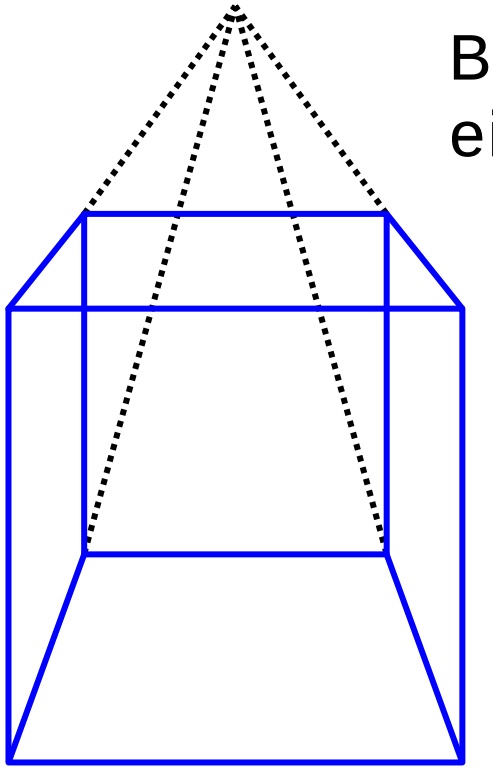


- Zentralprojektion:
  Augenpunkt im endlichen Abstand



- Parallelprojektion:
  Augenpunkt im Unendlichen

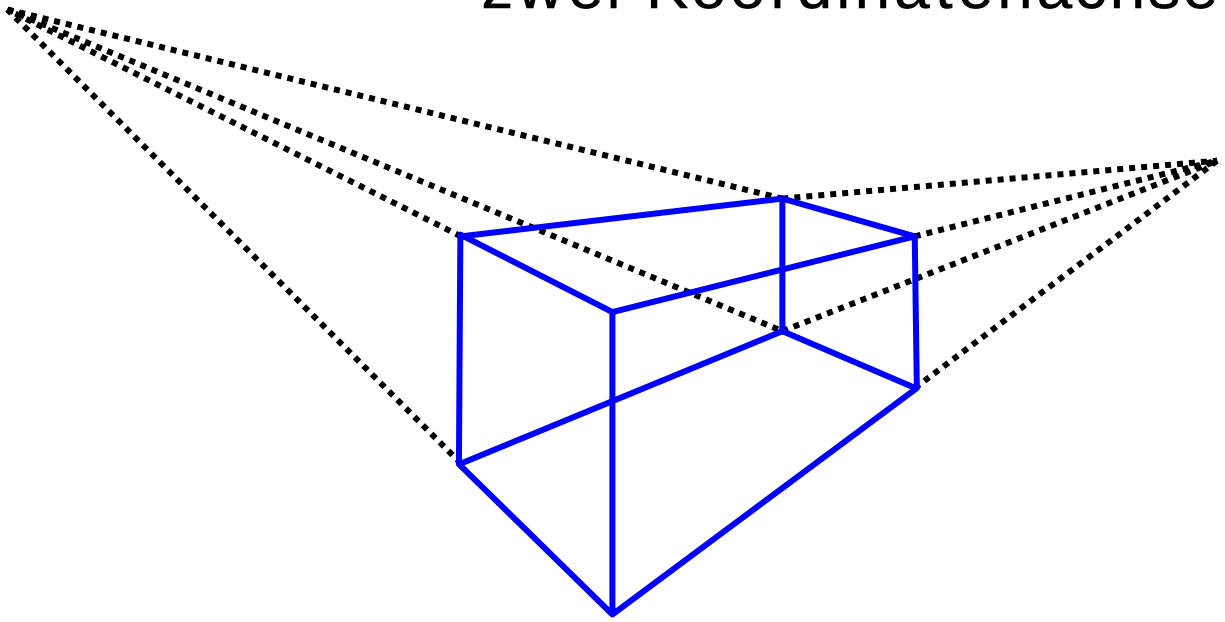
1 Fluchtpunkt



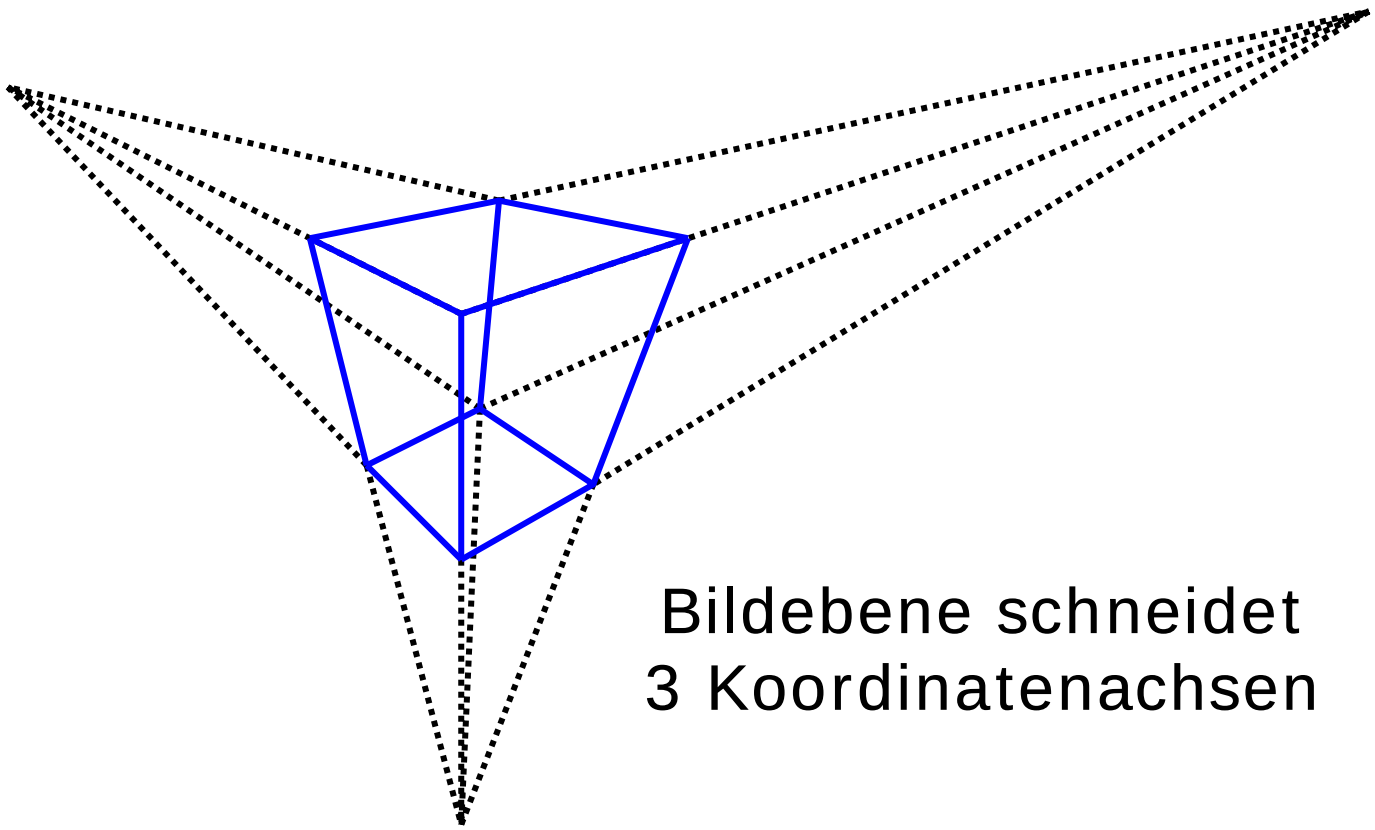
Bildebene schneidet
eine Koordinatenachse

2 Fluchtpunkte

Bildebene schneidet
zwei Koordinatenachsen



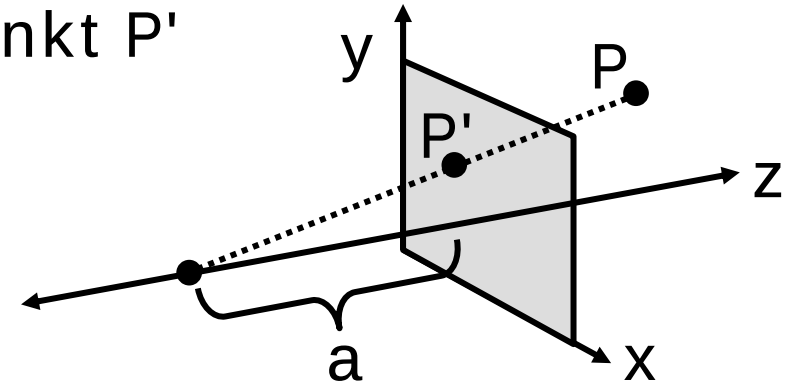
3 Fluchtpunkte



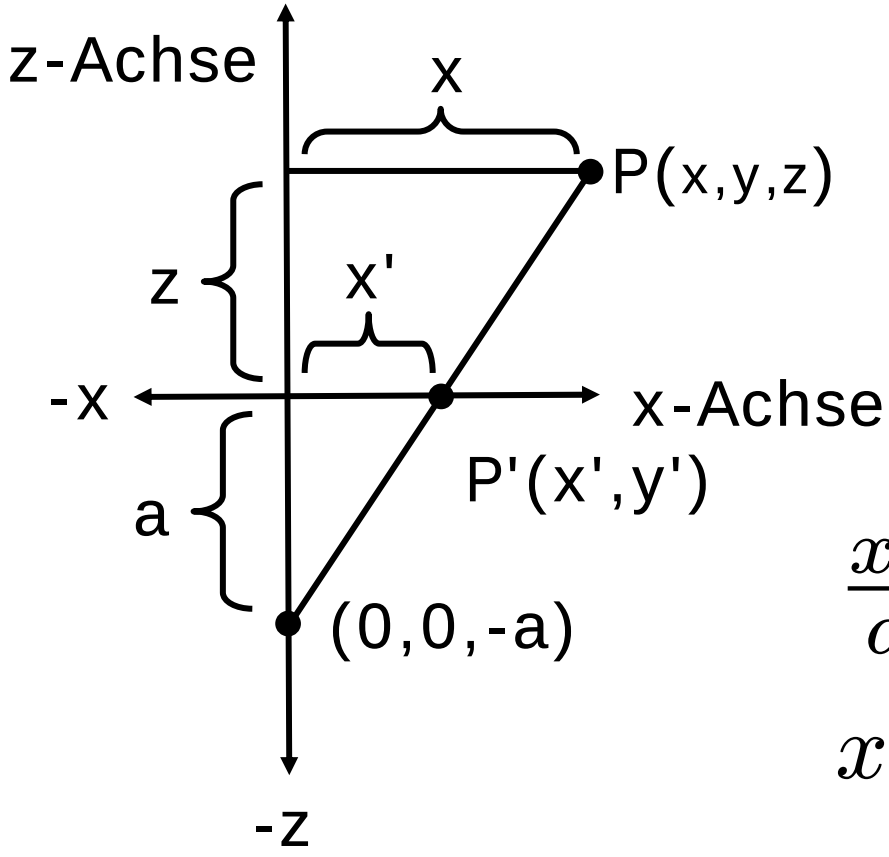
Bildebene schneidet
3 Koordinatenachsen

Aufgabenstellung
(im linkshändigen Koordiantensystem)

- Bildebene sei in xy-Ebene
- Augenpunkt sei auf negativer z-Achse bei -a
- Gegeben Punkt P
- Finde Schnittpunkt P'



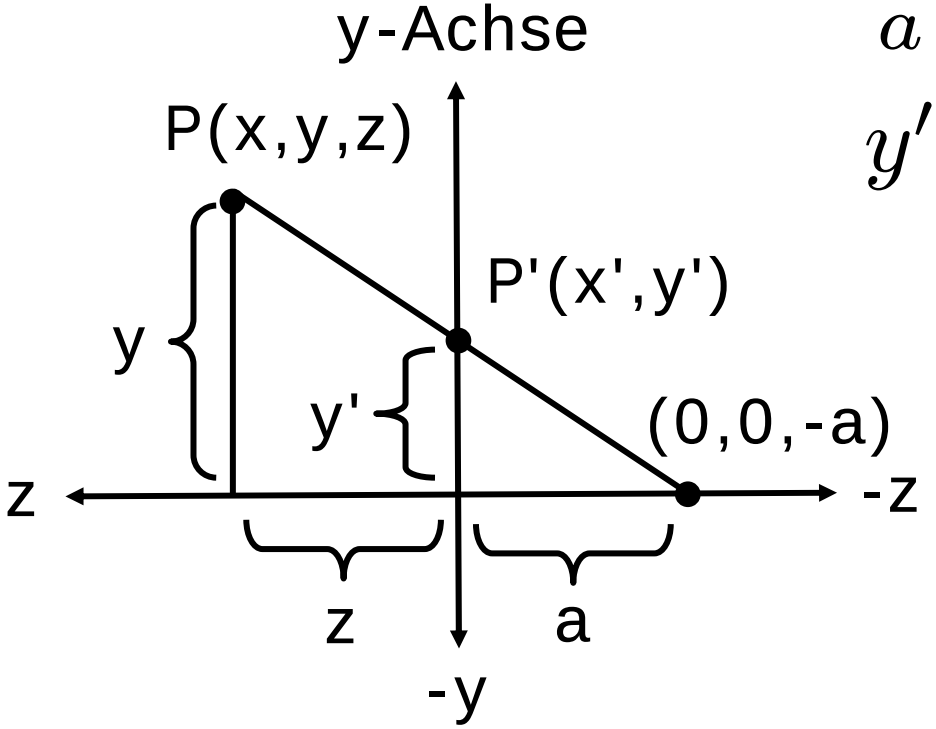
Blick von oben



$$\frac{x'}{a} = \frac{x}{a+z}$$

$$x' = \frac{x}{1+z/a}$$

Blick von der Seite



$$\frac{y'}{a} = \frac{y}{a+z}$$

$$y' = \frac{y}{1+z/a}$$

Ergebnis

$$x' = \frac{x}{1+z/a} \qquad y' = \frac{y}{1+z/a} \qquad \text{z merken}$$

$$x' = \frac{x}{w} \qquad y' = \frac{y}{w} \qquad w = 1 + z/a$$

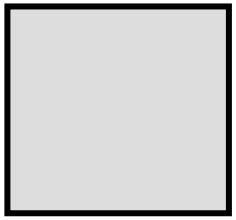
$$P' = (\tfrac{x}{w}, \tfrac{y}{w}, 0, 1) \;\; = (x,\ y,\ 0,\ 1 + z/a)$$

$$\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 1/a & 1 \end{pmatrix} \cdot \begin{pmatrix} x \\ y \\ z \\ 1 \end{pmatrix} = \begin{pmatrix} x \\ y \\ 0 \\ 1+z/a \end{pmatrix}$$

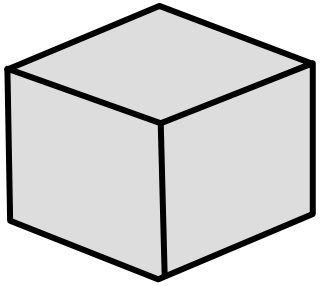
Parallelprojektion

Normalprojektion

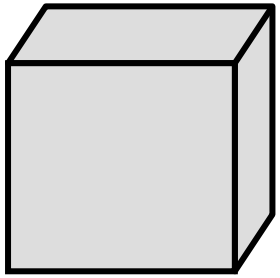
Grund-,Seiten-,Aufriss:



axonometrische Projektion:



schiefe Projektion

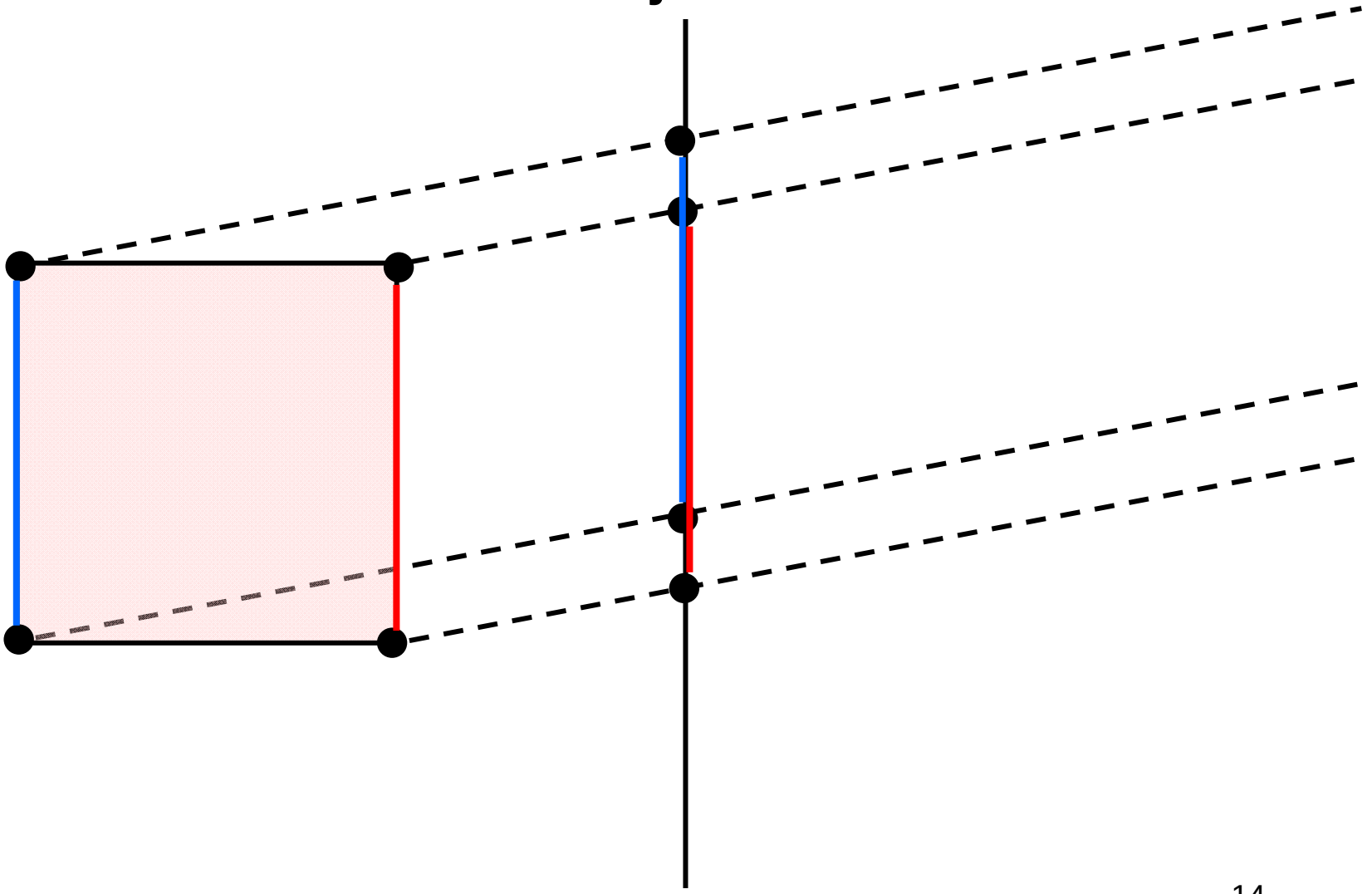


Normalprojektion

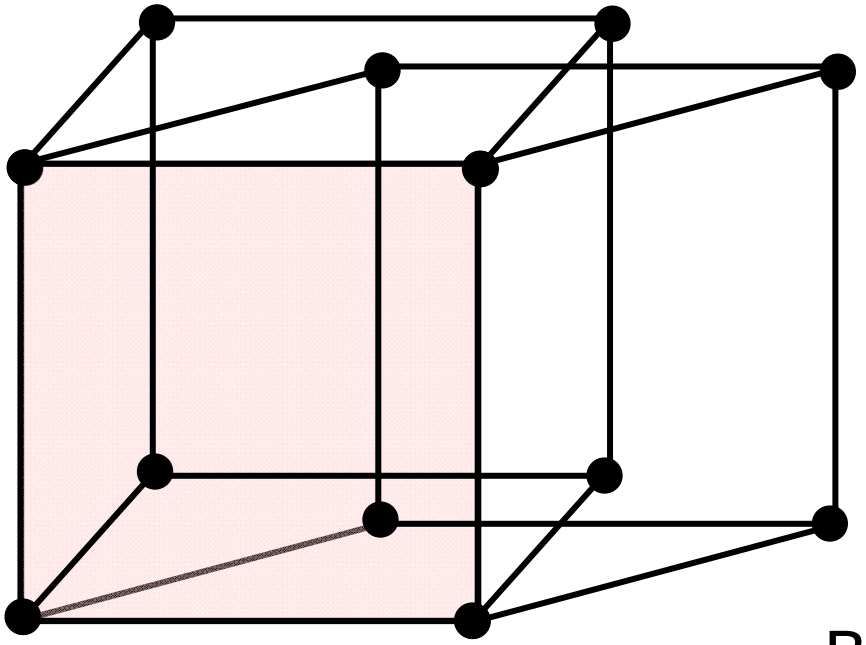
Bilde $(x,y,z,1)$ auf $(x,y,0,1)$ ab:

$$P_{ortho_{xy}} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

Schiefe Projektion



Schiefe Projektion



Beeinflusst durch ...

...Verkürzung in z-Richtung

... Anstellwinkel

Schiefe Projektion

$$x' = x - L \cdot \cos(\alpha)$$

$$y' = y + L \cdot \sin(\alpha)$$

$$z' = 0$$

$$(x - x')/L = \cos(\alpha)$$

$$(y' - y)/L = \sin(\alpha)$$

$$\tan(\beta) = z/L$$

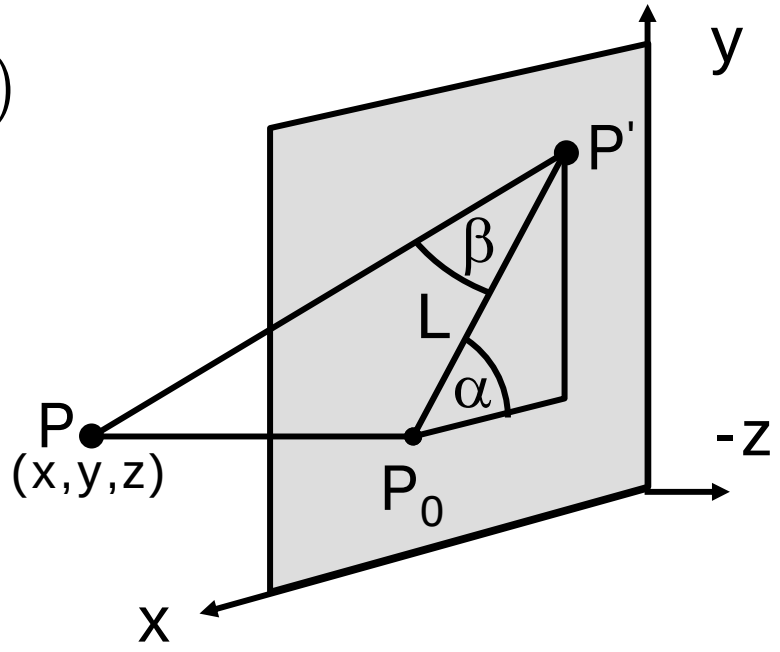
$$L = z/\tan(\beta)$$

$$x' = x - z \cdot (\cos\alpha)/\tan(\beta))$$

$$y' = y + z \cdot (\sin\alpha)/\tan(\beta))$$

α = Anstellwinkel

β = Verkürzungsfaktor



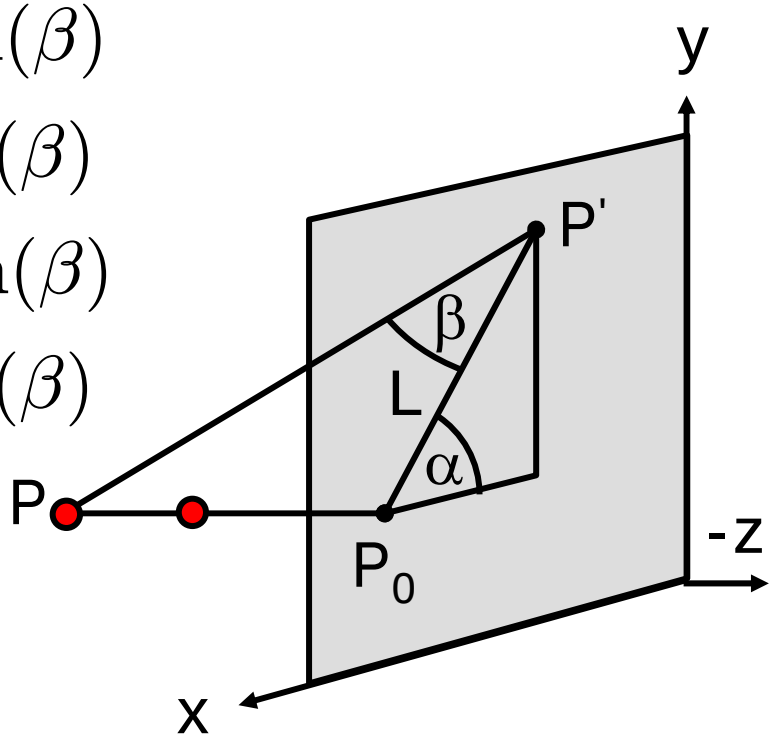
schiefe Transformationsmatrix

$$x' = x - z \cdot \frac{\cos(\alpha)}{\tan(\beta)}$$
$$y' = y + z \cdot \frac{\sin(\alpha)}{\tan(\beta)}$$
$$z' = 0$$
$$w' = 1$$

$$P_{schief_{xy}}(\alpha, \beta) = \begin{pmatrix} 1 & 0 & -\frac{\cos(\alpha)}{\tan(\beta)} & 0 \\ 0 & 1 & \frac{\sin(\alpha)}{\tan(\beta)} & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

x-Ausdehnung zu z-Ausdehnung

$$x'_1 = x_1 - z_1 \cdot \cos(\alpha) / \tan(\beta)$$
$$y'_1 = y_1 + z_1 \cdot \sin(\alpha) / \tan(\beta)$$
$$x'_2 = x_2 - z_2 \cdot \cos(\alpha) / \tan(\beta)$$
$$y'_2 = y_2 + z_2 \cdot \sin(\alpha) / \tan(\beta)$$



2 Punkte auf Lot zu x/y:

$$|x'_1 - x'_2| = |(z_1 - z_2) \cdot \cos(\alpha) / \tan(\beta)|$$
$$|y'_1 - y'_2| = |(z_1 - z_2) \cdot \sin(\alpha) / \tan(\beta)|$$

Verkürzungsfaktor

$$|P_1' - P_2'| = \sqrt{|x_1' - x_2'|^2 + |y_1' - y_2'|^2}$$

$$|P_1' - P_2'| = \sqrt{\frac{(z_1 - z_2)^2}{\tan^2(\beta)} \cdot (\cos^2(\alpha) + \sin^2(\alpha))}$$

$$\cos^2(\alpha) + \sin^2(\alpha) = 1$$

$$= \frac{z_1 - z_2}{\tan(\beta)}$$

$$d = \frac{1}{\tan(\beta)}$$

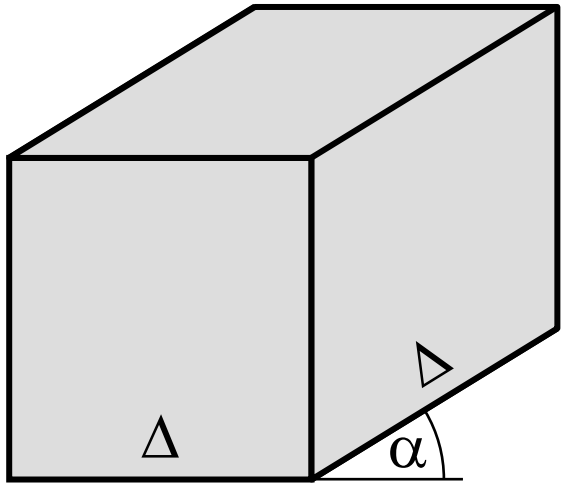
$\beta = 45° \quad \Rightarrow d = 1$

$\beta = 63.43° \quad \Rightarrow d = 0.5$

Beispiele für schiefe Projektion

$\beta = 45° \Rightarrow d = 1$

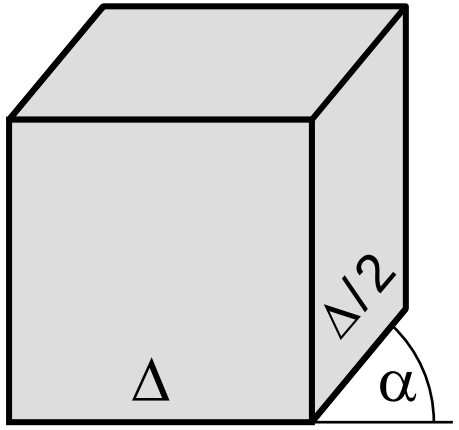
$\alpha = 35°$



Kavalierprojektion

$\beta = 63.43° \Rightarrow d = 0.5$

$\alpha = 50°$



Kabinettprojektion