Neuronale Netze (SS 2002), 3.7.

Fully recurrent neural networks

- **Definition:**

  Fully recurrent network: $(N, \rightarrow, \vec{w}, \vec{\theta}, \vec{f}, I, O)$ where $(N, \rightarrow)$ is a cyclic graph, often fully connected.

  Hopfieldnetwork: Fully connected with $I = O = N$, $f_i = H$, and the weight matrix is **symmetric**, i.e. $w_{ij} = w_{ji}$, $w_{ii} = 0$ for all i, j.

- General aim: networks as **associative memory**, i.e. given an exterior signal, the network relaxes to an associated memory state. Properties:
  - the pattern itself is the key,
  - memory is noise tolerant,
  - memory fills up 'smoothly'

- **Dynamics:**

  Asynchronous dynamics:

  $net_i(0) = o_i(0) = x_i$ for all neurons
  $net_i(t+1) = \sum_{j \rightarrow i} w_{ji} o_j(t) - \theta_i$
  $o_i(t+1) = o_i(t)$ for all $i \neq i_0$, i_0 chosen at random,
  $o_{i_0}(t+1) = H(net_{i_0}(t+1))$

  I.e. only one neuron i_0 is allowed to change the state.

  Alternatives:

  synchronous dynamics just like partially recurrent networks (problem: network might converge to cycles of length two instead of a stable state)

  continuous dynamics, modelled with differential equations, can be seen as the 'right' mathematical point of view which appropriately generalizes the discrete dynamics

- A state $\vec{o}$ of the neurons is **stable** iff $o_i = H(net_i)$ for all neurons i.

- **Energy function:** $E(\vec{o}) = -\frac{1}{2} \sum_{ij} w_{ij} o_i o_j + \sum \theta_i o_i$

- **Theorem:** The energy function does not increase during computation. The Hopfield network always reaches a stable state. Stable states are local minima of the energy function (with maximum number of 1.)
- The same does **not** hold for asymmetric weights, connections to the neuron itself, synchronous update.
- **Hebb-learning:**

  Biases as on-neurons, pattern $\vec{x}_p \in \{-1, 1\}^N$ should be learned as stable pattern.

  One shot learning rule:

$$w_{ij} = \frac{1}{N} \sum_p x_i^p x_j^p$$

- Hebb learning yields to stable patterns if the patterns are orthogonal.

  In addition to the learned patterns, superpositions and spurious states might be stable states (with usually higher energy).
- For random patterns, the number of stable patterns is of the order $N/logN$.

  For sparse patterns, we have $N^{3/2}/logN$.