1 The Simplex Algorithm

C is a conjunction of equations; i, I, j, J, n, m are integers; a_{ij}, b_i, e_i, d_i are constants; f, t are linear expressions; c_1 \ldots c_n are equations; x_i, y_j are variables.

simplex_opt(C, f)
  let C be of the form c_1 \land \cdots \land c_n
  for each i \in \{1, \ldots, n\}
    let c_i be of the form x_i = b_i + \sum_{j=1}^m a_{ij} y_j
  endfor
  let f be of the form e + \sum_{j=1}^m d_j y_j

  % Choose variable y_J to become basic
  if for all j \in \{1, \ldots, m\} d_j \geq 0 then
    return ⟨true, C, f⟩
  endif

  choose J \in \{1, \ldots, m\} such that d_J < 0

  % Choose variable x_I to become non-basic
  if for all i \in \{1, \ldots, n\} a_{ij} \geq 0 then
    return ⟨false, C, f⟩
  endif

  choose I \in \{1, \ldots, n\} such that
  \[ \frac{-b_i}{a_{ij}} = \min \{ \frac{-b}{a_{ij}} | a_{ij} < 0 \text{ and } 1 \leq i \leq n \} \]
  t := \frac{x_I - b_i - \sum_{j=1, j \neq I}^m a_{ij} y_j}{a_{ij}}
  c_I := (y_J = t)
  replace y_J by t in f
  for each i \in \{1, \ldots, n\}
    if i \neq I then replace y_j by t in c_i endif
  endfor
  return simplex_opt(\land_{i=1}^n c_i, f).